

2DV HYDRODYNAMIC MODELING OF RESERVOIRS

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ABSTRACT

Recently, simulation of hydrodynamic and water quality variables of a water body with numerical models is a common approach. Increasing in computer power and accuracy of laboratory results for field data are two important factors in rapid increasing in using these models. In this study, non linear and unsteady equations in Cartesian co-ordinate system are splitted and solved with finite volume method in staggered grid. Considering non-hydrostatic pressure, artificial compressibility approach is used for coupling velocity and pressure. Calculation of water elevation depends on user. Choosing a coefficient determines rigidity or being free surface. The effects of *TDS* and temperature have been considered for evaluating stratification of reservoir.

1 INTRODUCTION

Numerical models are useful and efficient tools that help for prediction the effects of management measures. The speed of running of the model depends on computer power and calibration precision based on observation data.

Although two-dimensional laterally averaged models are efficient, they are not common in studies yet. Computational cost of this model is equal to pure two-dimensional model. On the other hand, the existence of width in equations that leads to considering variation of width in simulation is the advantage of these models.

Most of the current two-dimensional laterally averaged models in the literature are based on the assumption that the vertical acceleration is small in comparison with other acceleration terms in the governing equation. Thus, a hydrostatic pressure distribution is assumed in the vertical plane.

For many applications in shallow water flows, the use of a hydrostatic pressure assumption can produce reliable predictions of the velocity distribution. However, for a range of applications involving conditions, such as rapid bed slope changes, strong stratification or short period waves, etc., the assumption of a hydrostatic pressure distribution is no longer valid.

Decreasing in calculation time and obtaining water elevation after calculation of velocity and pressure fields, are two reasons that made hydrostatic assumption more

common but in this study, after modification of velocities, according to inflow and outflow of each water column, the water elevation is obtained. In addition, there are some efficient algorithms for calculation of parameters of this approach, that leads to solving time problem [3].

2 GOVERNING EQUATIONS

The governing equations of two-dimensional flow in a vertical plane (i.e. 2DV) can be derived by integration of the momentum equation and continuity equation over a unit width:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uw}{\partial z} = \frac{-1}{\rho} \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left(2\nu_t \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial z} \left(\nu_t \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \right) \quad (2)$$

$$\frac{\partial u}{\partial t} + \frac{\partial w^2}{\partial z} + \frac{\partial uw}{\partial x} = \frac{-1}{\rho} \frac{\partial p}{\partial z} + \frac{\partial}{\partial x} \left(\nu_t \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) \right) + \frac{\partial}{\partial z} \left(2\nu_t \frac{\partial w}{\partial z} \right) - g \quad (3)$$

Where u , w are velocity components in x and z directions, respectively, P , ρ and g are total pressure, density and gravity, respectively and ν_t is eddy viscosity.

Momentum equation does not depend on the pressure, also pressure has been divided into two parts that named as: "hydrostatic pressure" ($\rho_0 g z$) and "access pressure" ($\rho_0 P$).

$$P = -\rho_0 g z + \rho_0 P^* \quad (4)$$

$$\frac{\partial P}{\partial x} = \rho_0 \frac{\partial P^*}{\partial x} \quad \frac{\partial P}{\partial z} = -\rho_0 g + \rho_0 \frac{\partial P^*}{\partial z} \quad (5)$$

Integrating Navier-Stokes equations over a unit width and substituting equations 5 in equation 2 and 3 lead to new form of momentum and continuity equations.

$$\frac{\partial Bu}{\partial x} + \frac{\partial Bw}{\partial z} = 0 \quad (6)$$

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x} (Bu^2) + \frac{\partial}{\partial z} (Buw) + \frac{\rho_0}{\rho} \frac{\partial P^*}{\partial x} = \frac{\partial}{\partial x} \left(2B\nu_t \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial z} \left(B\nu_t \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \right) \quad (7)$$

$$\frac{\partial w}{\partial t} + \frac{\partial}{\partial x} (Buw) + \frac{\partial}{\partial z} (Bw^2) + \frac{\rho_0}{\rho} \frac{\partial P^*}{\partial z} = \frac{\partial}{\partial x} \left(B\nu_t \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \right) + \frac{\partial}{\partial z} \left(2B\nu_t \frac{\partial w}{\partial z} \right) - \frac{\rho - \rho_0}{\rho} g \quad (8)$$

Approximations that are used to simplify governing equations are as follow:

- Boussinesq approximation: based on such approximation, water acts as an incompressible fluid, so density is independent of pressure and $(\rho^{-1} D\rho / Dt)$ term at Navier-Stokes equation is negligible. So conservation of mass equation converts to conservation of volume and this simplification leads to equation 6.

- Averaging over width: The improved model in this study is laterally averaged. The basic assumption in these models is that width variation is negligible in comparison with two other dimensions. Accordingly the continuity equation in y direction converts to constant pressure in this direction.

3 IMPROVEMENT OF NUMERICAL MODEL

Due to presence of nonlinear terms in continuity equation, there is no analytical solution for them. Therefore the numerical solution is used. Numerical solution of nonlinear differential equation in case of converting them to linear algebra equations is possible in several methods including method of characteristics, finite difference method, finite element method, finite volume method, spectral method and boundary element method. Considering many advantages of finite volume method for instance conservation in every volume control, physical understanding ability of discrete equation and desirable results make this method popular during the last decade.

4 COMPUTATIONAL GRID SYSTEM

Selecting coordinate system in numerical method is an important factor. Although Cartesian system has some limitations in case of irregular geometries and especially in calculating wall shear stress with high accuracy, regarding considerable reduction of calculations complexity, this system is applied in many cases. Easy and accurate implementation of boundary conditions is another advantage of Cartesian system. Position of variables of this model is presented in Figure 1.

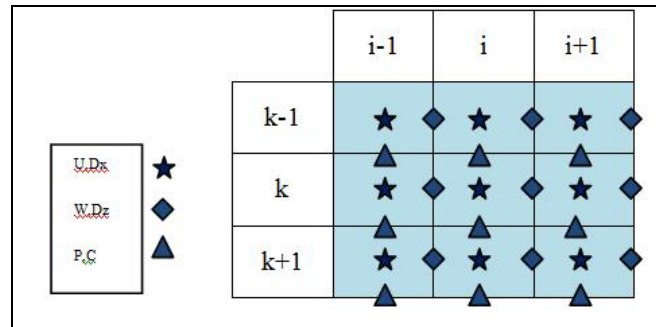


Figure 1. computational grid system and position of variables

4.1 Splitting the equations

In order to use time splitting method for solving governing equations, splitting the continuity equation is essential. Accordingly, first of all the transport equation is solved in four steps and transport of u and w variables is calculated in horizontal and vertical directions. One dimensional calculation in every step is one of the advantages of time splitting method.

5 CALCULATION OF PRESSURE

As pressure is absent in continuity equation and is not coupled with velocity in non-hydrostatic condition, continuity equation needs some alteration in order to find a relationship between pressure in continuity and momentum equations. Several approaches can be used for this purpose. Pressure-based method (PBM) and artificial compressibility method (ACM) have been widely used in researches. In this model ACM has been used. Applying the “artificial compressibility” approach first proposed by Chorin [1968], in which the derivative of the pressure in time appears in the continuity equation. In this technique the artificial pressure created by non-zero mass balance in each cell propagates in all directions, similar to a surface wave during an iterative procedure to get the continuity equation satisfied [5]. Finally continuity equation converts to eq.(9).

$$a \frac{\partial p^*}{\partial t} + \frac{\partial BU}{\partial x} + \frac{\partial BW}{\partial z} = 0 \quad (9)$$

a, is the artificial compressibility coefficient that changes upon the conditions.

Substituting continuity equation in momentum equation leads to Poisson equation. This equation is a relationship between pressure of one cell and four neighboring cells. Consequently Tomas algorithm coefficients obtain and pressure at each time step is calculated [6].

Imposing known boundary conditions in Tomas coefficients has two states.

- Known velocity of upstream or downstream
- Known pressure of upstream or downstream

Regarding velocity-pressure coupling, velocity modification is required after calculating pressure at every time step.

6 MODIFICATION OF WATER ELEVATION

Water elevation can be simulated both in the form of rigid or free surface. After modification of velocities for every water column, considering inflow and outflow of each column, variation of water elevation calculates and finally the model corrects elevation.

7 TEST CASE

A Plexiglas channel was filled with fresh water, of depth $h=10\text{cm}$, and then a wooden gate was placed in the channel, at a distance $l=30\text{cm}$ from the left end. Household salt was added to the water behind the gate until a stable density of $\rho = 1011.4 \text{ kg/m}^3$ (equivalent to 16.3 kg/m^3 of salinity) was obtained. The gate was then suddenly removed.

The computational domain of length 3 m was divided into 20 and 150 cells in the vertical and horizontal directions respectively, with corresponding space intervals of $dx=0.02\text{m}$ and $dz=0.005\text{m}$. The simulation time and time intervals were chosen to be as 20 s and 0.05 s , respectively. The model results consisting of the salinity distributions and current patterns, together with the corresponding photographs have been plotted in figure 2 for 4.4, 6.8, 9.7 and 17.5 seconds after removal of the gate [5].

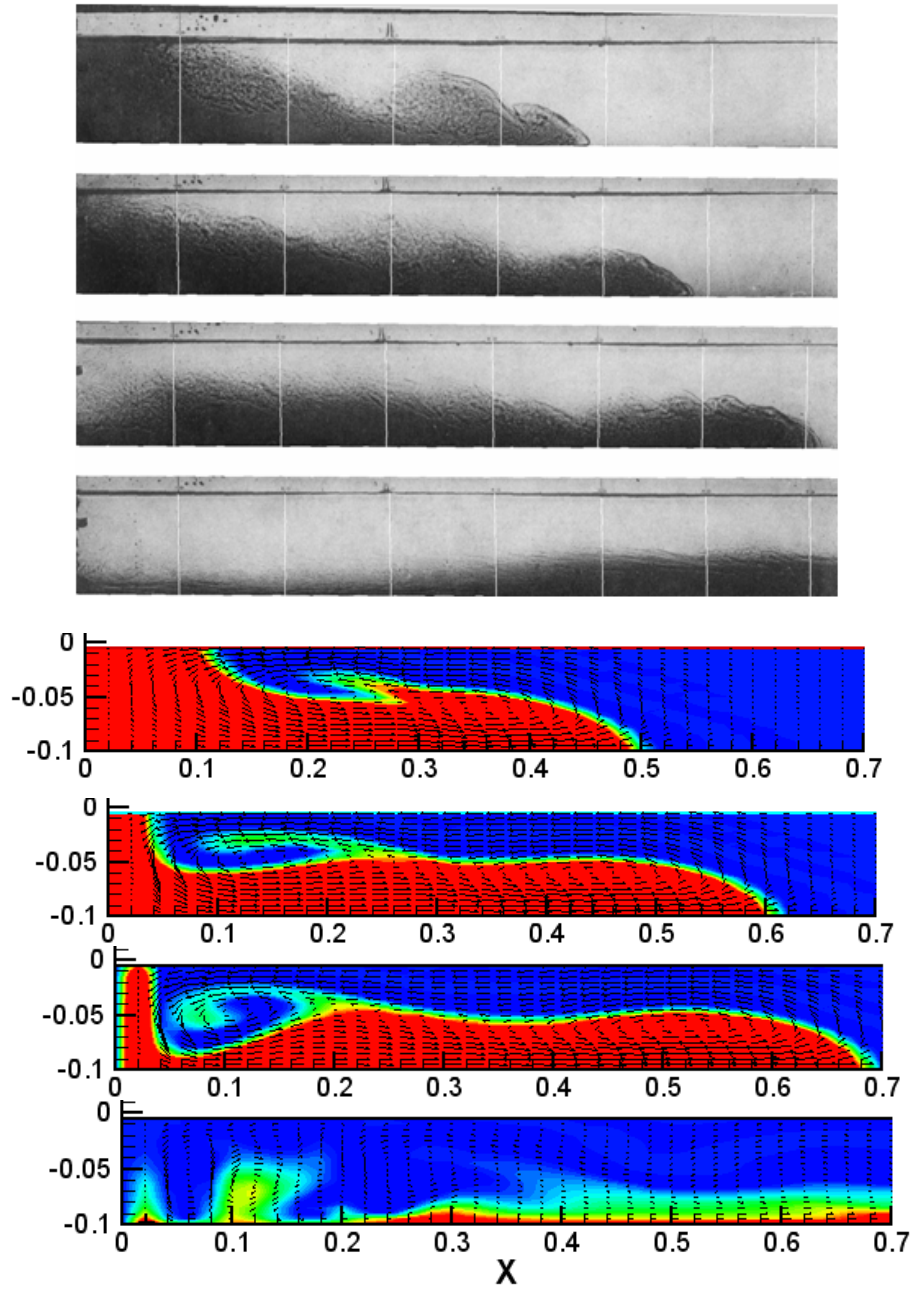


Figure 2. Salinity distributions approximately 4.4, 6.8, 9.7 and 17 seconds after removing the gate, (a) laboratory experiments photographs and (b) numerical results containing velocity fields.

8 RESULT

In this study, a two dimensional laterally averaged model with non-hydrostatic assumption has been improved for reservoirs and shallow waterbodies. Cartesian coordinate system has been used. Non-linear differential equations have been solved using finite volume approach. Time splitting method is used for Separation of equations. For coupling velocity and pressure, artificial compressibility method is applied. Entering a coefficient by user determines rigidity or being free surface of waterbody. Experimental data verified the model results.

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