

WATER QUALITY MODELS FOR RIVERS AND STREAMS. STATE OF THE ART AND FUTURE PERSPECTIVES

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ABSTRACT

Mathematical models for water quality problems in rivers and streams are now an important tool in water resources management. Their basic principle is the differential equation of pollution transport, which can be integrated both in analytical and numerical form. From this equation the broad category of “fickian” models has been developed. The numerical procedures available so far are the finite difference and the finite element method. More recently the finite volume method has been proposed. The majority of practical cases consider the one-dimension approach, very significant in large rivers; but a better simulation of the real behaviour requests, at least, a two-dimension approach. The discharge of hot water is a frequent occurrence that gives rise to the thermal pollution. The water pollution problem in rivers and streams involve economical, social and political impact, for which the programming models can be very useful. To be efficient in the largest set of practical cases the model can benefit from sensitivity and uncertainty analysis. Research is now in progress aiming at the improvement the fickian models and at the identification of other kinds of models able to better understand the complex phenomena of pollution transport in rivers and streams.

1 INTRODUCTION

The progress in computer technology and in the mathematical procedures has introduced some tools that are now essential also in order to approach the water quality problems. These tools are the mathematical models, which act as a representation of the reality and allow its problems to be handled without interfering directly with the reality. Verifying indeed a solution directly in a natural entity requires costly and complex engineering interventions, often destructive. Vice versa, if the model represents correctly the reality and all its relevant phenomena, a solution can be examined in a short time and at a much lower cost.

Reviewing the peculiarities of the river water quality models some fundamental aspects should be recalled. In the next paragraphs these aspects will be examined, focussing, in particular, on the most advanced contribution that the science and the practice have been able to produce in the last years.

The numerous applications, in many practical cases around the world, as appreciable in the technical and scientific literature, confirm that one of the first points to be recalled is the fact that the mathematical models are not an abstract entity, but they

request, first of all, proper data to be put into work. Data collection and making them available are a very important chapter in model application. Moreover, any concern about data implies the highest professional expertise. Therefore, as a general statement, the mathematical models must be viewed in a close relation with the availability of good and significant data, to be handled by selected and responsible people.

Coming now to the water quality models for rivers and streams, their fundamental source has to be recalled, namely the advection-dispersion differential equation, which is inferred from the well known “fickian approach”

$$\frac{\partial C}{\partial t} = - \left(\frac{\partial v_x C}{\partial x} + \frac{\partial v_y C}{\partial y} + \frac{\partial v_z C}{\partial z} \right) + E \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right) \pm S - KC \quad (1)$$

in which $C = C(x,y,z;t)$ is the pollutant concentration, E is the dispersion coefficient, $S = S(x,y,z;t)$ the external contribution and K the reaction coefficient. The final goal of expression (1) is therefore the pollutant concentration in the whole water body, at a certain instant t .

Obviously, in a quite general view, other sources can be considered, leading to other kinds of models, but the approach based on the advection and dispersion processes makes up the broadest category of “fickian” water quality models.

A lot has been written about this approach, and this matter is largely acquainted in the hands of scientists and field engineers. The following notes will therefore pay more attention to the more recent contributions that have made the models easy to be applied, stressing their power as a reliable tool for the main problems in water resources management.

The application of river water quality models requires the knowledge of the velocity field, at any instant and any point of the water body. For this purpose the river hydraulics proposes the De Saint Venant equations, the solution of which requires appropriate mathematical procedures. The reality of a polluted river is so complex, that to try and search for a detailed set of information in the water body can be put in an order higher than that practically achievable. Collecting data in a river suffers from imprecision and a dispersion characterises the usable values. This identifies a level of precision that can be obtained only according to the sensitivity and the practical experience of the operator. Once more we must say that the fate of the models depends on the people who utilise them.

Following these considerations, we can say also that the knowledge of the velocity field can be accepted with some approximation, without entering the complex machinery of the De Saint Venant equations, taking the opportunity given by simpler procedures, which, supported by experimental measures, enable to identify the river hydraulic characteristics in a satisfactory way. On the other hand, we must remember that at the present time there are available refined computing packages that, beside the evolution of the fundamental equation of pollutant transport, include also the identification of the velocity field by means of precise procedures.

2 FORMAL DEVELOPMENTS

The fundamental differential equation (1) is an interpretation of the pollution transport in an infinitesimal volume of the water body. Its main concepts need to be brought to the entire mass of the river. In other words, the equation must be integrated.

An analytical integration in the most general, three-dimension and time-dependent case is not easy. Burdensome mathematical manipulations are necessary. Vice versa, the one-dimension case, in which all the terms are supposed constant in the y and z directions and depend only on the longitudinal co-ordinate x , is much simpler and is now universally recognised. In practice, this case approaches the behaviour of a stream or a long river stretch, in which the quality characteristics can be considered constant in any cross-section, but variable along the stretches into which the water body can be split. The constant values in a cross-section normally come from integration along the vertical direction z and assuming an average value along the transversal direction y .

The analytical integration leads to the well known Runkel formulation, which is currently applied and is very useful to focus on the role of the basic terms involved in the process of pollution transport, like the water velocity, the dispersion coefficient and the reaction coefficient.

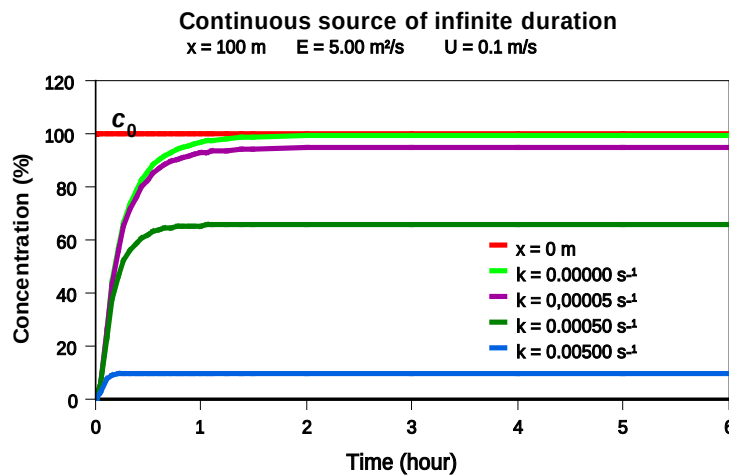


Figure 1. Concentration of a non-conservative pollutant for different values of the reaction coefficient.

Fig. 1 shows how the model can predict the pollutant concentration in a river cross-section downstream from the injection point, as a function of time and for different values of the reaction coefficient.

The formulation can consider both steady and unsteady conditions. For more complex problems, like, for instance, a “polluting wave” varying continuously in a time interval, it implies complicated adjustments that make the application uneasy.

During the last decades the progress in numerical calculus has promoted the development of numerical procedures, which have been intensively applied to the

fundamental equation of pollutant transport. The process starts from transforming the equation into a “discrete” expression.

In the numerical approach we should clearly emphasise that the expression contains two terms that behave in a different way according to the basic mathematical outlines, namely a “hyperbolic” term, determined by the discretisation of the first order derivative of the pollutant concentration, and the “parabolic” term coming from the second order derivative. These two terms follow different way of proceeding to be done one another independently.

In the discretisation process the general differential equation of pollutant transport is replaced by an expression of finite terms, both in space, with intervals of prefixed size, Δx , Δy and Δz , along the coordinate directions, and in time, with a Δt interval. Normally the integration of the fundamental equation is brought to the solution of linear equations, or linear equation systems, with many steps and variables, and involves the broad chapter of linear systems mathematics.

At the present time the numerical methods can be grouped into two broad categories, namely the finite difference method (FDM) and the finite element method (FEM).

The finite difference method is the simplest and most original way of solving a differential equation. Unfortunately the adoption of discontinuous finite terms does not necessarily maintain all the characteristics of the original differential expression, because they cannot take into account all the details that only a continuous approach can appreciate. Moreover, even under the mathematical viewpoint there are aspects that cannot be handled without constraints and calculation difficulties. As a matter of fact, the solution is always approximate.

Following a more philosophical consideration we may say that the true solution is in practice meaningless, as all the approaches have to pay for some approximation. Consequently, the solution of a discrete interpretation is currently characterised by a discretisation error (or truncation error), very difficult to quantify. An appreciation of such an error can only be made after comparing several solutions obtained by means of different procedures and reflects, implicitly, the statement that “a discontinuous step cannot be viewed as a finite part of the infinite.”

Essential for the application of the finite difference method is the way the reality, namely the water body, is decomposed in finite steps with respect to space and time. These steps have their own peculiarities and, consequently, they lead sometime to solutions that cannot be accepted, being very far from a real interpretation of the phenomenon under examination

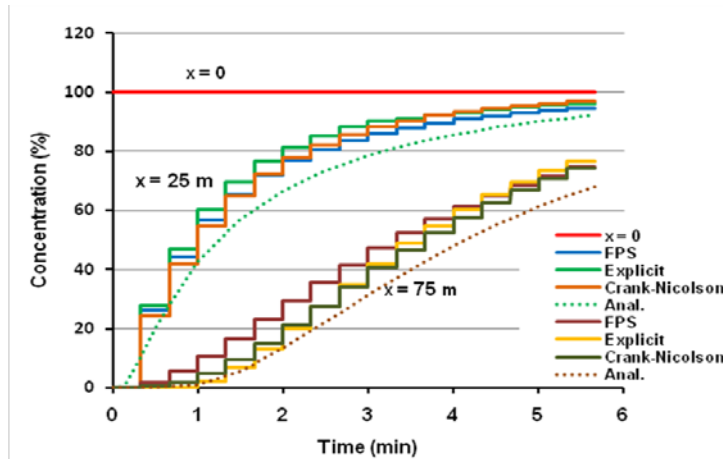


Figure 2. Different solutions for a continuous injection of infinite duration, adopting several numerical procedures, and comparison with an analytical solution.

There are several procedures for the achievement of a final expression applicable in practice to a river stretch. Just to mention, the Crank-Nicolson, the “explicit” and the “FPS” are the most common procedures for the application of the finite difference method.

Because every procedure starts from a particular transformation of a continuous reality into discrete terms, it is reasonable to expect different results. Fig. 2 shows a comparison among various solutions for some significant cross sections of the river, downstream from the point of pollutant injection. The solutions are in form of “steps” corresponding to any time interval into which the equation is discretised. In the same figure also the corresponding analytical solution is shown, in form of a continuous dotted curve.

The discrepancies shown in the figure are relatively small and are expected to be of the same order of magnitude as the experimental values that can be observed in a river. Adopting smaller time and space intervals the discrepancies could vanish and the final result will be closer to that of the analytical approach. Obviously, decreasing the size of the time and space intervals inevitably increases the computational burden. A reasonable compromise is therefore necessary in the discretisation process.

The progress in computer technology and in numerical calculus has favoured the development of the finite element method, originally applied for mechanical structures. It can provide interesting solutions also in water quality modelling. Like the finite difference, the crucial point of the finite element method is the risk of instability due to the simultaneous presence of parabolic and hyperbolic terms in the fundamental equation. Several procedures have been tried to interpret separately the advection and the dispersion pollutant transport.

Also in this case it is of interest to compare this solution with that of an analytical procedure, as shown in Fig. 3 for a non-persistent pollutant.

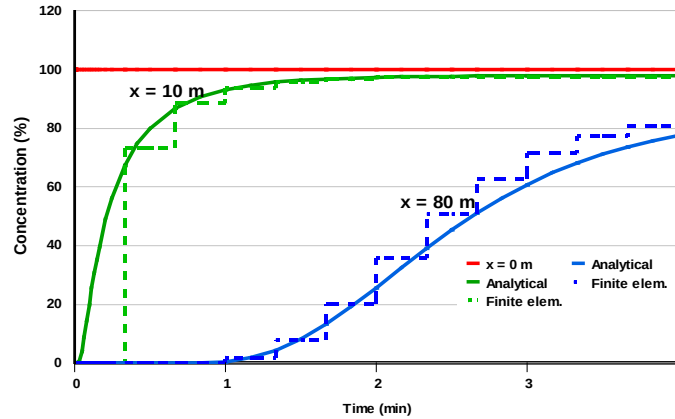


Figure 3. Solution for the injection of a non-persistent pollutant, at two cross-sections of the river.

In the figure the pollutant behaviour for two river cross-section ($x = 10$ m and $x = 80$ m), as solution of the finite element method, is compared with the analytical solution. It is worthy to notice that the finite element method gives values that match up to those given by the analytical method, but as far as the distance from the injection increases there is a clear trend for the final element to anticipate the value of the resulting pollutant concentration.

It is worthy to stress that the numerical methods have gained in validity in comparison to the analytical procedures for an application to more complex realities. The finite element method can help to face more complex problems, like, for instance the pollutant injection in form of “wave” at the initial cross-section of the channel. Another important privilege of the method is that it can be adapted to complex river geometry, because the elementary intervals can assume a whatsoever form and size, obviously at the expenses of more burdensome calculations.

Fig.4 shows the deformation of a “wave” injected at the cross-section $x = 0$ and the progressive abatement of the peak, as far as we proceed downstream in the river.

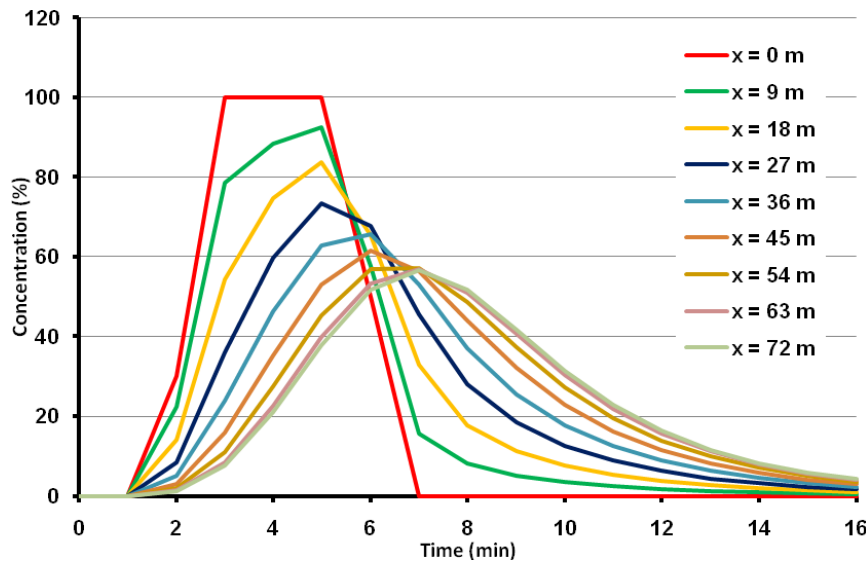


Figure 4. Deformation of a “pollution wave” as a function of time, for eight element of a river..

The finite element method is now one of the most advanced tools to solve differential equations in discrete form. It can be adapted to the most common cases of river pollution and can give a significant response. The initial steps, when the mathematical fundamentals have to be understood and tailored to the reality of the water body, can appear somewhat difficult, but as soon as the algorithm has been translated into a form suitable to be filled with the relevant data, the finite element method is able to give prompt solutions.

Like for the finite difference method, the application of the finite element has to comply with some restrictions involving the basic data, which, aggregated in significant expressions, must satisfy severe constraints. The usable data must therefore remain inside pre-established intervals. A stable and acceptable solution can be achieved only if Δt and Δx are sufficiently small. Such a condition is not always consistent with a river reality, in which a very short reach is difficult to identify and can lose significance in comparison with the size of the other entities that characterise the water body.

Compared with the finite difference method, the finite element method appears more difficult to handle, but the positive aspects of the latter can be appreciated in the most complex cases, when an application of the finite difference could request several repeated equations, difficult to handle even with the most advanced computing facilities.

In more recent time a new promising tool has been proposed, beside the finite difference and finite element: the finite volume method. It is applied also for the numerical integration of the fundamental differential equation of pollutant transport. Accordingly, the partial differential equation is discretised into cells, inside of which the pollutant concentration is conveniently evaluated. The variation of the concentration is identified in the cell boundary, with the assumption that the value lost by a cell is gained

by the contiguous one. Proper analytical transformations provide to structure the algorithm into a linear equation system, to be solved by means of the conventional procedures. The cells can have one or higher dimension, then the method is applicable to problems of different realities.

3 THE THERMAL POLLUTION

Very frequent is now the case in which a river receives the discharge of hot water, coming from a thermal electricity plant or a chemical process. Having a density lesser than that of the receiving body, the heated water gives rise to the density currents, very often appreciable in form of a thermal plume. Moreover, the increased temperature in the water body alters the living conditions of aquatic weeds and animals. This is, in short, the thermal pollution, which is expected to increase in the near future owing to the growth of thermal power plants, which need huge amounts of water for cooling.

In a river stretch, assuming the water temperature, τ , as significant indicator of the thermal pollution, the fundamental equation of pollutant transport (1), in its three-dimension form, becomes

$$\frac{\partial \tau}{\partial t} + \left(\frac{\partial v_x \tau}{\partial x} + \frac{\partial v_y \tau}{\partial y} + \frac{\partial v_z \tau}{\partial z} \right) = E \left(\frac{\partial^2 \tau}{\partial x^2} + \frac{\partial^2 \tau}{\partial y^2} + \frac{\partial^2 \tau}{\partial z^2} \right) \pm \mathfrak{S} \quad (2)$$

where $\mathfrak{S}$ denotes the effect of heat exchange between the river and its environment. Such an exchange is due primarily to man-made activities with the persuasion that the great quantity of colder water running in the river can extinguish the heat, contributing to the environmental benefit. Such a persuasion is not entirely correct. Moreover, there are several other causes of heat injection, due to natural phenomena, that should not be neglected in the thermal balance of the river.

In a long river, the effect of a heat injection is appreciated on a long stretch. A one dimension approach is therefore useful, assuming that the heat effect is expressed by the average temperature on the cross-section.

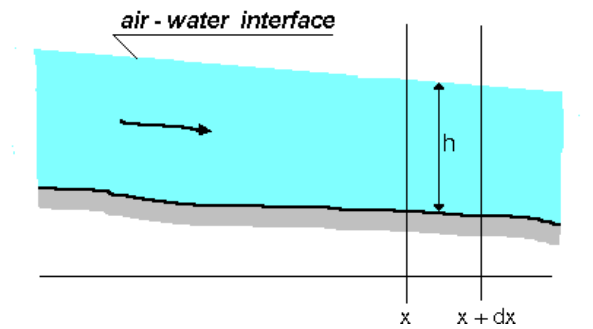


Figure 5. The elementary particle in the one-dimension approach, for thermal pollution simulation.

For this purpose it is convenient to identify the elementary particle of the water body

as in Fig. 5, extended to the entire depth and width of the stream, between the two cross-sections at x and $x + dx$, with average depth h .

The typical case of discharging cooling water from a thermal power station or from a chemical plant is sketched in Fig. 6. In this case the flow in the receiving body increases by the amount of discharged hot water, which sometime can be of the same order of that in the river. The water velocity in the river and the other hydraulic characteristics must be conveniently adjusted.

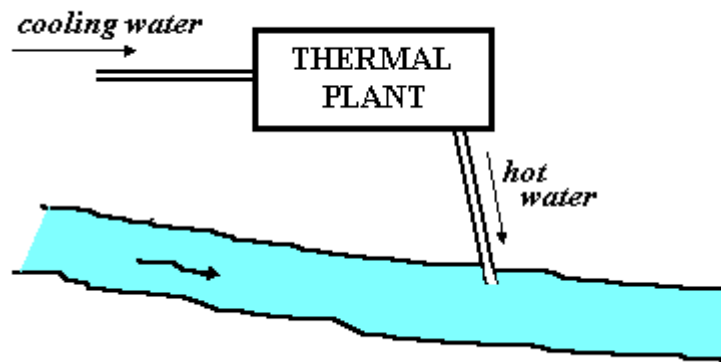


Figure 6. Injection of the hot water discharged by a cooling system.

Particularly in large rivers, a considerable heat can be exchanged with the air, through the free surface of the stream. Significant can be also the exchange through the river bottom and the embankments, which can have a proper temperature due to geological and climatic conditions, varying according to the season, and depending on the latitude. Another possible heat exchange is with the sediments transported by the stream and eventually settling on the bottom.

Although important for the overall thermal balance, someone of these forms of heat exchange can be neglected in the practical cases, being confined in particular scientific speculations. The problem is then restricted to the effect of a concentrated discharge of hot water.

After manipulating the equation (2) according to the mathematical procedures described in the preceding chapters, we can examine the behaviour of water temperature, obtaining an interesting information for a correct protection of the river water.

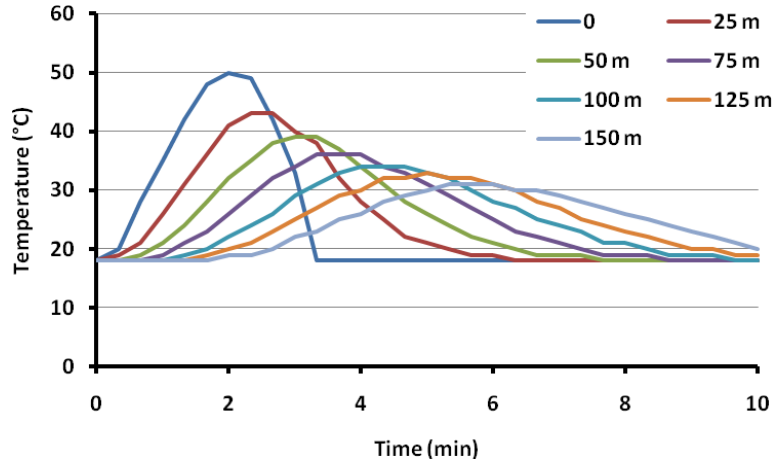


Figure 7. Temperature profile of the channel, after the end of heat injection with an initial temperature of 18 °C.

Fig. 7 shows an example of hot water discharge into a river in which the initial water temperature was already matching the natural environmental conditions ($\tau = 18^\circ\text{C}$), as a function of time, at different cross-sections.

4 THE MULTI-DIMENSION APPROACH

The one-dimension approach, adopted in the majority of practical cases, cannot be always considered in practice. In large rivers an injected pollutant requires some time before reaching an acceptable uniform distribution on the cross-section and the pollutant presence is appreciated in form of a two- or three-dimension plume. A multi-dimension approach can better simulate the river behaviour. In the majority of practical cases a two-dimension approach can give acceptable results. In such a condition the fundamental differential equation of pollution transport (1), for a persistent pollutant ($K = 0$), becomes

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = E \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) \quad (3)$$

in which u and v are the water velocity components along, respectively, the x and y axes.

An analytical solution of equation (3) requires several mathematical manipulations, with complex formulations. A numerical procedure is therefore more convenient. An application of the finite difference or finite element method can be easy in principle. Nevertheless, several severe conditions for the usable data are necessary, in order to avoid unacceptable solutions.

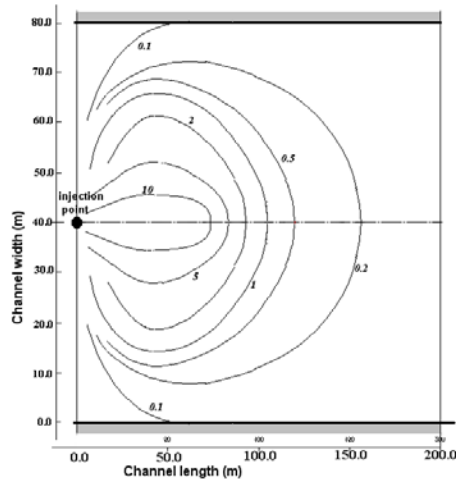


Figure 8. The response of a two-dimension model in form of a pollutant plume.

As shown in Fig. 8, the solution of a two-dimension model can be plotted in form of a plume, by means of isoconcentration curves. Proper software packages can help to present the model output directly in graphical form.

5 OPTIMISATION MODELS

The complexity of water problems requires more tools to focus on the role that a clean river plays in the management of an integrated water resources system. An acceptable pollutant concentration in a river is the consequence of interventions involving all the contaminating sources. The reduction of the pollution load requests to control not only the urban discharge, but also the industrial and agricultural activities, together with the adoption of suitable treatment processes. All these actions have economical, social and political impact, which the optimisation models can help to understand. Optimisation models are therefore a powerful tool in the hand of the people who have responsibility to intervene and decide how to utilise the available resources

The optimisation (or programming) models belong to a very large chapter of mathematics, described in numerous textbooks and experienced in many practical applications. These models are able to identify, for a given input, an output value that can be considered, within a certain respect, the best of all the other possible ones.

The substantial point of an optimisation model is the identification of an objective suitable to be translated into mathematical terms (objective function). This function is accompanied by relationships stating the way in which the variables are one another connected in the problem (constraints). It is worthy to mention that the optimisation models are the essential tool for all the situations in which a conflict arises among the problem variables.

Proper mathematical procedures are available for the development of the programming models. An important type of these models is characterised by having both the objective function and the constraints in form of linear functions of the variables. This is the linear programming model, which belongs to a very effective

branch of applied mathematics, especially utilised in the field of water resources problems.

The following example illustrates how an optimisation model can be used in a general context of the water quality problems in rivers. As shown in Fig. 9 the river stretch receives the discharge of a sewage, which pollutes the downstream reaches. The application of one of the simulation models described in the preceding chapters allows the water quality in all the river cross sections to be quantified.

To reduce the pollution level, we can build a suitable wastewater treatment plant applying the most appropriate technology, which involves a proper cost. Anyhow, a plant able to treat the whole amount of wastewater could be too costly, and a reasonable alternative is the possibility to discharge part of polluting wastewater directly in the river, on the assumption that the natural behaviour of the stream will reduce the pollutant concentration in the downstream reaches

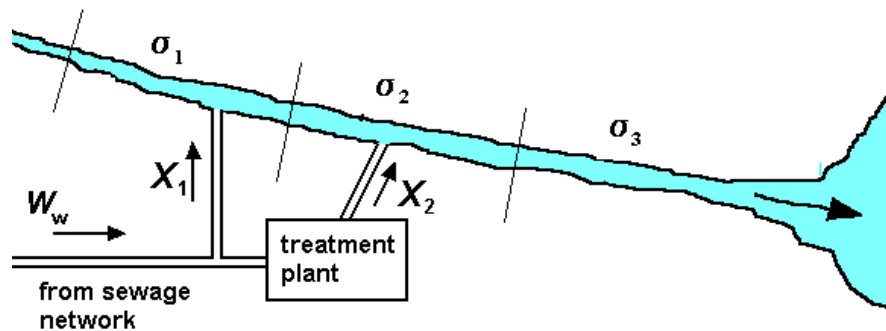


Figure 9. A simple case for the application of linear programming model.

A polluted river is the cause of reduced possibilities for other forms of water utilisation and we must take into consideration a relevant cost, which, although difficult to quantify, is eventually proportional to the amount of the wastewater directly discharged in the river. We identify thus two variables, namely the volume of wastewater destined to the treatment plant and that directly discharged. These two variables are included in the objective function and the solution of the programming model gives an “optimal” combination of them.

To improve the result, the post-optimal analysis can help to determine the most reliable value of the single variables and to assess how any variable can affect the behaviour of the whole system under examination.

In a general management of a river, the common practice tells us that the complexity of the problem cannot be fully analysed by means of a single model. Several models are therefore necessary, one another interacting, in order to better appreciate the various aspects of the reality under examination. In such a framework, we can organise a combination of simulation and optimisation models as sketched in Fig. 10.

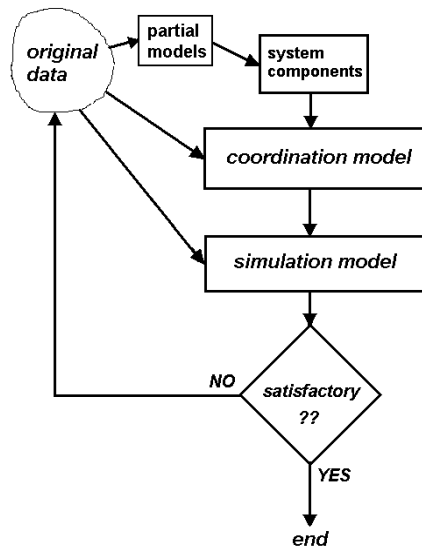


Figure 10. Role and combination of several models for water resources management.

The model combination makes up an iterative process. If the reality is expected to act too many times in a wrong way, some components can be modified, adopting new operation rules and changing the size of the most important works. The process can be repeated until a satisfactory result is achieved.

6 MODEL CALIBRATION AND VALIDATION

The set of data used for the construction of a model is normally limited to particular aspects of the reality. Similarly, the user's familiarity with the phenomenon, acquired in previous experience, does not cover necessarily the entire domain in which the problem is considered. Moreover, the necessity to comply with the formal rules imposed by the available mathematical procedures imposes simplifications and escapes that cannot always match the reality of the problem to be investigated. There is, therefore, the possibility that the model is not able to give a response entirely acceptable.

After the model has been constructed and run for the first times, it is necessary to test it with the whole set of available data, proceeding to the calibration, which a thorough process as sketched in Fig. 11.

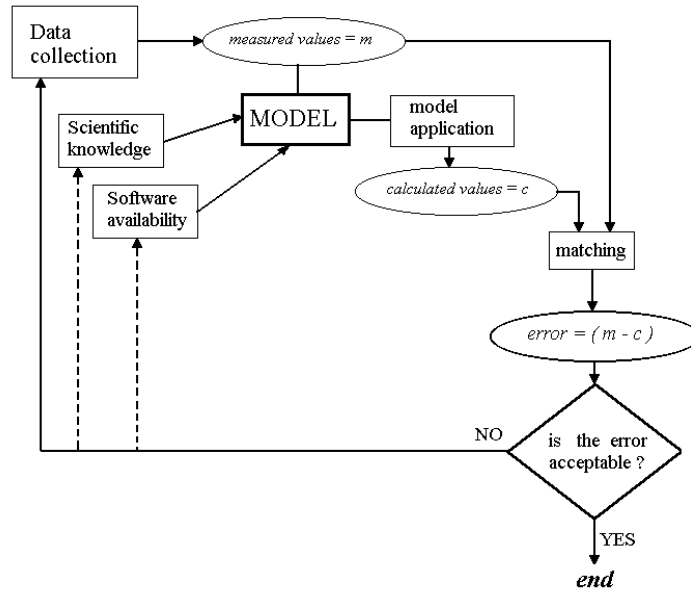


Figure 11. Calibration of the mathematical model.

The calibration is based on the error, namely the difference between the calculated and measured values of most significant terms.

Due to the overall complexity of the machinery, there is a little probability that the error could be zero, even in the best working conditions. It is therefore necessary to adopt a suitable range of acceptability of the error, with a pre-established threshold, trespassing which the result of the model cannot be accepted. If, for a given range of acceptability, as sketched in the Figure, the error can be accepted, the model is considered satisfactory. Vice versa, if the error exceeds the range, the model must be revised, modifying some components in a repeated iterative process. If necessary, new experimental data have to be collected.

An immediate way to revise the model is to change the invariant terms, namely the constants and the parameters. Normally, if all the assumptions are correct, a few repeating runs are sufficient, but, in case the error keeps being unacceptable, some basic assumption should be revised, including the mathematical procedure.

Even though satisfactorily calibrated, some aspects still remain restricted to the particular reality on which the model has been developed. These aspects are tied to the rest of the reality by means of constraints that could not be taken into a correct consideration at the very initial step of the process, when the model was developed. Consequently, there is another set of factors that could cause the model to give wrong, or not significant, results.

It is then necessary to test the model having in front, as far as possible, the entire reality in which the problem is inserted, following a procedure known as validation. This entails a broader set of data, perhaps the comparison with other models and the commitment of an expertise sometimes different from that employed for the construction of the model.

The model validation is particularly relevant to the process of decision making, when the response of the model is used to support an intervention on the reality, which could be expensive with undesirable consequences. The decisions are in the hands of people which should have attention to several aspects of the problem that are not necessarily based on physical, biological and engineering expertise, but include economics, social and political knowledge. Therefore, the results of a model application have to be inserted in the best possible frame of a true interpretation of the reality and require to be presented in a way easy to be understood by persons having various experience and culture.

7 SENSITIVITY AND UNCERTAINTY ANALYSIS

Even though the model has been successfully calibrated and validated, a doubt still remains about its capability of giving reliable results when it is currently applied to the largest set of practical cases. Any application is conditioned by the value of some terms that normally cannot be measured correctly or are assumed a priori, starting from exogenous aspects.

Such considerations lead to the sensitivity analysis, by means of which the model is run repeatedly after assuming several values for the parameters that make up its internal structure, according to the scheme of Fig. 12.

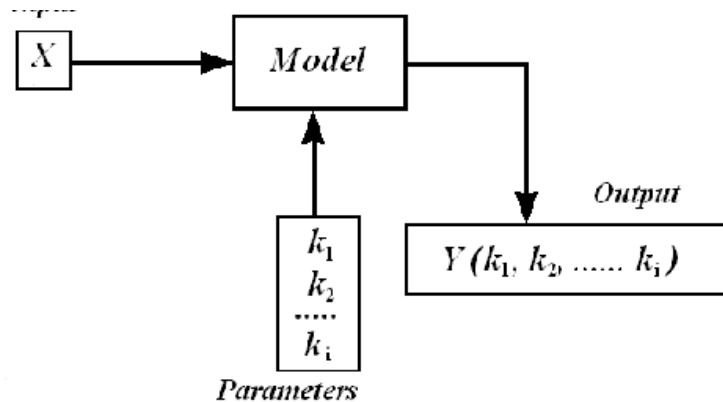


Figure 12. The effect of parameters variability.

For a given input, the model produces an output that depends on the specific value assumed for its parameters. The output, consequently, has also its variability range, which, moreover, depends on the inner mechanisms of the mathematical formulations. It may happen that parameters having a limited range of variability cause an output with a large range: the model is therefore highly sensitive. Vice versa, if parameters with a great variability range give an output with a narrow range, the model is “good” and reliable.

There are several ways of conducting the sensitivity analysis. All the ways require repeated model runs assuming well defined values of one or several more significant parameters. The simplest and most immediate procedure consists of comparing the results of several model runs after assuming the largest set of parameter values,

compatible with the real aspects of the river and the pollution discharge.

All the model components can be characterised by imprecise values that can give an output not completely reliable. In fact, both the input variables and the parameters can be affected by imprecision and measurement errors. Moreover, the model itself is based on approximations and mathematical developments that do not interpret correctly the reality. Therefore a more accurate form of analysis has been developed, namely the uncertainty analysis.

While the sensitivity analysis is based on the assumption of arbitrary values for some model components, the uncertainty analysis takes into account that all the components are characterised by uncertain values and have an impact on the final result.

The uncertainty analysis becomes essential if the water quality model is itself the component of a more complex system made up by several models one another interacting. Typical is the interconnection of a water quality model with a hydrological model, which provides the input values in terms of water velocity, or the case in which the water quality model provides the basic data for models aiming at a rational use of water resources.

The uncertainty analysis works according to the scheme of Fig. 13. It starts with the identification of the error for the various model components. Because every measured value has its own error, suitable tools are necessary in order to perform a thorough analysis.

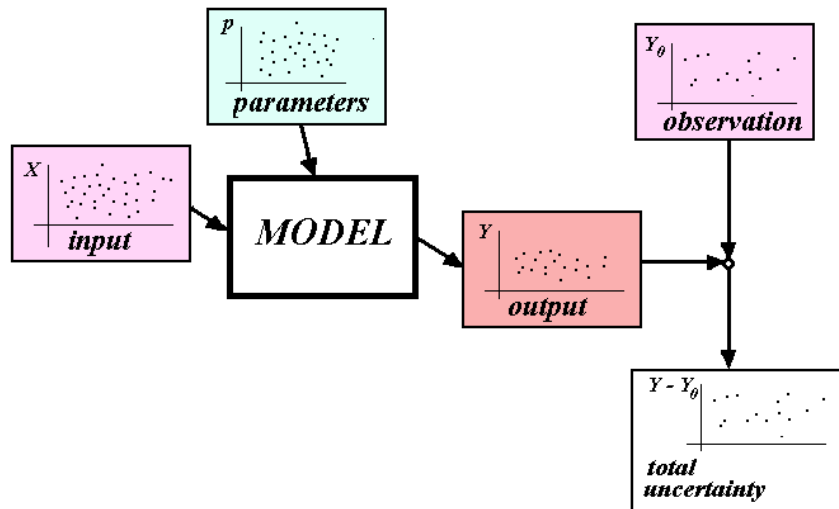


Figure 13. The various sources of uncertainty for the model.

Statistics can help to perform the analysis and, among its tools, the Monte Carlo method can be very useful, which allows an accurate insight on the peculiarities of the values to be used, focussing on the errors that can be done during the measure. Statistical methods can provide other procedures to answer the primary question about the model capability to correctly simulate the reality.

8 NEW TRENDS AND FUTURE DEVELOPMENTS

The main researchers' interest is now concentrated in making the model a good machinery that works promptly and is able to give a correct picture of the reality in practical applications and in the hands of experienced professionals. A lot of time is spent for the mathematical procedures in the numerical field. A suitable integration form of the fundamental differential equation of pollutant transport still remains the main goal. The proposed numerical methods have to be adapted to the multi-dimension cases.

The fickian approach is still the scientific background for the simulation of pollution transport in rivers and channels, but some in-field observations prove that the pollutant in a river does not entirely comply with an advection-dispersion interpretation. One of the most interesting attempts to update the fickian approach is the use of the fuzzy numbers, borrowed from a theory now widely applied to the frequent cases characterised by imprecise or short information.

Flourishing innovative researches have now given rise to the family of non-fickian models, which can lead to formulations easily applicable with high level of reliability.

Among these models very promising are those based on the random walk theory, now currently applied to practical cases characterised by uncertainty. According to such theory, the motion of a pollutant particle is interpreted as a sequence of random "jumps" with different length and duration.

Another interesting approach is based on the theory of fractals that belongs to the chapters of statistics and mathematics dealing with the chaotic systems. Advanced research activities involve methods and tools of the artificial intelligence, like the expert systems. The artificial neural networks and the genetic algorithms are also considered for the purpose.

The water quality problems in rivers and artificial channels remain one of the most compulsory questions in water resources management and the mathematical models are the most effective tool to understand the behaviour of pollutants. The success of mathematical models relies on the way the real and man induced processes are understood and formally interpreted. This can be done only with a skilled capability, essential not only to develop the model, but also to appreciate the results of an application and put them into practice. In such a way the mathematical model for river water quality can give a very good contribution to the enhancement of the expertise that is necessary in the daily practice.