

MONTHLY MONSOON RAINFALL FORECASTING USING REGRESSION AND ARTIFICIAL NEURAL NETWORK MODELS

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ABSTRACT

Forecasting of monsoon rainfall has been one of the most important tasks in water resources development and management. Indian agriculture sector heavily depends on monsoon rainfall for successful harvesting. In the past, prediction of rainfall was mainly performed using regression models, which provide reasonable accuracy in the modelling and forecasting of complex physical systems. Alternatively, Artificial Neural Networks (ANNs) have been proposed as efficient tools for modelling and forecasting. The objective of this study is to investigate the use of ANNs in forecasting of monsoon rainfall in the North Central region of India. A feed-forward multi-layer perceptron type of ANN architecture trained using the popular back-propagation training algorithm was employed in this study. Other techniques investigated for modeling monthly monsoon rainfall include regression and naive models for comparison purposes. Both linear and non-linear regression models were considered. The data employed in this study include monthly rainfall and monthly average of the daily maximum temperature in the North Central region in India. Specifically, four regression models, two naive models, and one ANN model have been developed. The performance of various models was evaluated using a wide variety of standard statistical parameters and graphical aides. The results obtained in this study for forecasting monsoon rainfalls for North Central region of India have been encouraging. India's economy and agricultural activities can be effectively managed with the help of the availability of the accurate monsoon rainfall forecasts.

1 INTRODUCTION

Water is a scarce natural resource that is available to the human beings on the earth. The availability of water is not uniform across the globe. The water resources in India are highly variable with respect to both time and space. That is, we experience floods and droughts at the same time in different parts of the country and the availability of water is concentrated during monsoon months in India and not throughout the year. Indian agriculture sector heavily depends on monsoon rainfall for successful harvesting. The majority of rainfall in most of the country is concentrated in the four monsoon months, namely, June, July, August, and September (JJAS). The accurate estimates of the forecasted monsoon rainfalls are needed in the proper planning of the agricultural,

industrial, and municipal water activities in the country.

Rainfall is an extremely complex, non-linear, and dynamic process that is not clearly understood and is difficult to model. The rainfall depends on many climatic variables such as temperature, relative humidity, wind speed and direction, atmospheric pressure in addition to its own value in the past. The various influencing variables mentioned above are highly inter-related among each other in a complex and non-linear manner. Forecasting of monsoon rainfall, therefore, has been one of the most important tasks in water resource development and management and research. The monsoon rainfall forecasting in India is currently carried out using conventional methods of regression models. Alternatively, Artificial Neural Networks (ANNs) can be used as efficient tools for modelling and forecasting of engineering and natural systems.

Rainfall modeling is an extremely complex and difficult problem involving many variables which are interconnected in a very complicated manner. The forecasting of monsoon rainfall in India started more than a century ago (Blanford 1884; Walker 1908, 1933). Since then, the forecasting of monsoon rainfall has received much attention of the researchers in India. Singh et al. (1995) presented a regression model to forecast All India summer monsoon rainfall using average sea pressure and heat energy data. Thapliyal (1997) presented a very nice description of various methods adopted for monsoon rainfall modelling in India. They presented four different models, namely, parametric models, power regression, multiple regression, and dynamic stochastic transfer function models. Sahai et al. (2003) developed multiple regression models using fourteen different global oceanic climatic variables to forecast all India monsoon rainfall. Iyengar and Kanth (2003) employed time series analysis methods to model all India and regional rainfall data and found that their non-linear time series models were able to explain only about 50% of the variability in the rainfall data. Some use of ANNs has also been found to model monsoon rainfall in India. Sahai et al. (2000) compared time series and ANN models to forecast all India summer monsoon rainfall and found ANNs to be reasonable in forecasting monsoon rainfall in India. Recently, Kumar et al. (2007) used ANN and genetic programming techniques to model regional monsoon rainfall data of Orissa using global tele-connection climatic data. However, the efforts on the use of ANNs for monsoon rainfall forecasting have rather been limited.

The objective of this study is to investigate the use of ANNs in forecasting of monsoon rainfall in the North Central region of India. A feed-forward multi-layer perceptron type of ANN architecture trained using the popular back-propagation training algorithm was employed in this study. Other techniques investigated for modeling monthly monsoon rainfall in India include regression and naive models for comparison purposes. Both linear and non-linear regression models are developed. The data employed in this study include monthly rainfall and monthly average of the daily maximum temperature in the North Central region in India. Specifically, four regression models, two naive models, and one ANN model have been developed for each monsoon month. The performance of various models was evaluated using a wide variety of standard statistical parameters and graphical aides. This paper starts by presenting a brief overview of the Indian monsoon and the study area and data. The model development is taken up next before presenting the discussion of results. The last section summarizes the paper in the form of conclusions and future scope for further research work in the closely related areas.

2 INDIAN MONSOON AND STUDY AREA AND DATA

The monsoon arrives in India two branches: the Bay of Bengal branch and the Arabian Sea branch. The monsoon typically breaks over Indian Territory by around 25 May each year, when it lashes the Andaman and Nicobar Islands in the Bay of Bengal. It strikes the Indian mainland around 1 June near the Malabar Coast of Kerala. By 9 June, it reaches Mumbai; it appears over Delhi by 29 June. The Bay of Bengal branch, which initially tracks the Coromandal Coast northeast from Cape Comorin to Orissa, swerves to the northwest towards the Indo-Gangetic Plain. The Arabian Sea branch moves northeast towards the Himalayas. By the first week of July, the entire country experiences monsoon rain, on average, South India receives more rainfall than North India. However, Northeast India receives the most precipitation. Monsoon clouds begin retreating from North India by the end of August; it withdraws from Mumbai by 5 October. As India further cools during September, the southwest monsoon weakens. The monsoon usually withdraws from the country by about November.

Data employed in this study were taken from the Indian Institute of Tropical Meteorology (IITM) web-site (<http://www.tropmet.res.in>). The IITM has divided the whole India into seven homogeneous regions i.e. Western Himalayas (WH), the Northwest India (NWI), the North Central India (NCI), the Northeast India (NEI), the Interior Peninsular India (IPI), the East Coast of India (ECI) the West Coast of India (WCI) as shown in Figure 1. The monthly, seasonal, and annual rainfall and temperature data for the period 1901-2003 is freely available at IITM web site (<http://www.tropmet.res.in>) for the seven homogeneous regions and all India. In this study, we focussed on one specific Indian region i.e. North Central India (NCI) region. Separate models have been developed for the four monsoon months June, July, August, and September. The data derived from the NCI region have been employed to train and test all the models in this study. A schematic of the NCI region is shown in Figure 2. The data used in this study include monthly rainfall (mm) and temperature (°C) of NCI region which includes five states i.e. Delhi, Rajasthan, Uttar Pradesh, Madhya Pradesh and Maharashtra (see Figure 2).

The total length of the rainfall and temperature data of 103 years (1901-2003) is available for NCI region. The data for the year 1901-1952 were used for training/calibration and the data for the period 1953- 2003 were used for testing all models. Descriptive statistics for rainfall and temperature data of all the months which are used in this study are shown in Table 1 and Table 2, respectively. It is clear from Table 1 that July is the rainiest month in NCI region with maximum rainfall being the highest. The variation in rainfall for the month of July is highest while April rainfall data are the most skewed. Also, the month of June experiences least amount rainfall among the monsoon months. Table 2 shows that May is the warmest month while other months are comparable. The development of various models is presented next.

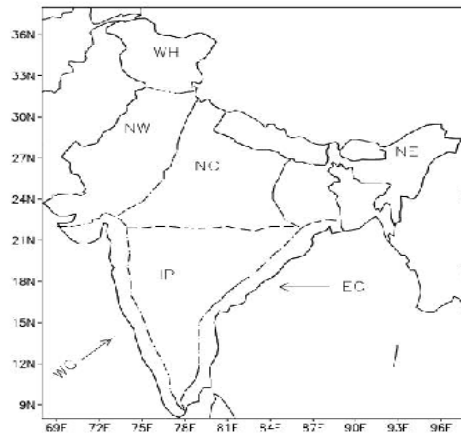


Figure 1: Homogeneous monsoon regions of India (Source: <http://www.tropmet.res.in>)



Figure 2: Study area for rainfall forecasting, North Central India. (Source: <http://www.tropmet.res.in>)

Table 1: Descriptive Statistics for rainfall data

During Training								
Statistics	March	April	May	June	July	August	September	JJAS
Minimum	0.1	0.4	6.8	35.0	132.5	198.7	92.5	833.5
Maximum	63.7	84.6	92.9	295.1	510.3	444.5	309.6	1369.7
Mean	16.1	15.8	35.9	159.0	339.2	333.7	210.4	1042.5
Standard Deviation	15.8	14.4	20.5	68.8	78.2	50.2	60.2	117.4
Skewness	1.4	2.5	1.0	0.2	-0.5	-0.5	-0.2	0.5
During Testing								
Minimum	0.1	1.3	3.8	61.5	188.2	151.6	73.2	668.1
Maximum	48.2	67.1	78.9	307.1	464.2	442.5	338.9	1315.7
Mean	13.8	15.3	37.4	156.3	319.8	311.8	223.7	1011.6
Standard Deviation	12.3	11.3	20.2	59.0	66.5	53.1	59.5	127.8
Skewness	1.1	2.0	0.3	0.4	0.1	-0.2	-0.3	-0.1

Table 2: Descriptive Statistics for temperature data

During Training							
Statistics	March	April	May	June	July	August	September
Minimum	28.1	33.3	35.1	31.9	29.2	28.7	29.3
Maximum	33.9	38.8	41.6	36.8	33.9	31.4	32.6
Mean	31.2	36.4	39.0	34.4	31.1	29.9	30.8
Standard Deviation	1.5	1.4	1.2	1.1	1.1	0.5	0.9
Skewness	-0.2	-0.4	-1.1	-0.1	1.1	0.3	0.5
During Testing							
Minimum	29.1	34.7	36.1	32.1	29.7	29.3	29.7
Maximum	34.9	40.0	41.1	36.7	34.5	31.7	33.4
Mean	31.7	37.0	39.2	34.6	31.6	30.3	31.0
Standard Deviation	1.4	1.1	1.0	0.9	1.0	0.5	0.8
Skewness	0.2	0.4	-0.7	-0.3	0.6	0.6	1.0

3 MODEL DEVELOPMENT

Three types of models were developed, naïve models, regression models, and ANN models. Two naïve models were considered as the bench mark models. The first one was a persistence model in which the forecasted value of a variable is taken as the value in the recent time step; whereas, the second naïve model was the mean model in which the forecasted rainfall in a particular month was taken as the mean rainfall. The performance of all the models developed in this study was evaluated using a wide variety of standard error statistics. These include Nash-Sutcliff coefficient of efficiency, Pearson coefficient of correlation, average absolute relative error (AARE), normalized mean bias error (NMBE), normalized root mean square error (NRMSE), and threshold statistics (TS). In addition, the Akaike Information Criterion (AIC) and mean square error (MSE) were employed to determine the optimal ANN architecture in this study. A description of these statistics may be found in Jain and Srinivasulu (2004).

3.1 Regression Model Development

Among regression models, both linear and non-linear structures were considered, specifically, one multiple linear regression model (MLRM) and three multiple non-linear regression models (MNLRMs) were developed. The first non-linear multiple regression model (MNLRM1) was a quadratic polynomial of all the input variables, the second non-linear multiple regression model (MNLRM2) was a cubic polynomial of all the input variables, and the third non-linear multiple regression model (MNLRM3) was a power regression model in terms of all the input variables. The structures of the four regression models are described by the following equations, respectively.

$$\text{MLRM:} \quad Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots \quad (1)$$

$$\text{MNLRM1:} \quad Y = \beta_0 + \beta_1 X_1 + \beta_2 X_1^2 + \beta_3 X_2 + \beta_4 X_2^2 + \dots \quad (2)$$

$$\text{MNLRM2:} \quad Y = \beta_0 + \beta_1 X_1 + \beta_2 X_1^2 + \beta_3 X_1^3 + \beta_4 X_2 + \beta_5 X_2^2 + \beta_6 X_2^3 \dots \quad (3)$$

$$\text{MNLRM3:} \quad Y = \beta_0 X_1^{\beta_1} X_2^{\beta_2} \dots \quad (4)$$

Where X 's are independent variables, Y is the dependent variable, and β 's are the regression coefficients to be determined. The values of the calibrated regression coefficients were determined using the software MS Excel in this study.

The first step in the development of any mathematical model is to determine the significant inputs for the model. In this study, the significant inputs were determined by calculating the cross-correlation coefficients and auto-correlation coefficients for the monthly temperature and rainfall data series, respectively. The results of this correlation analysis are presented in Table 3.

Table 3: Correlation Coefficient of $P(t)$ v/s other variables for various months

Variable	June	July	August	September
$P(t-1)$	0.2557	-0.0403	-0.0849	0.0942
$P(t-2)$	0.1048	0.2557	0.0403	0.0849
$P(t-3)$	-0.2038	0.1048	-0.2557	-0.0430
$T(t)$	-0.7980	-0.7814	-0.6943	-0.6524
$T(t-1)$	0.0324	-0.7980	-0.7814	0.6943
$T(t-2)$	0.1508	-0.0324	-0.7980	0.7814
$T(t-3)$	0.1083	0.0151	0.0324	0.7980

The values of the correlation coefficient for the input variables selected to model a particular month rainfall are presented in bold font in the above table. Thus, there were three input variables for modeling the monsoon rainfall for each of the months in the NCI region in India. The calibration data were used to calibrate all the regression

models and testing data set was used to test the performance of the models.

3.2 Artificial Neural Network Technique

An ANN is a highly interconnected network of many simple processing units called neurons, similar to the biological neurons in the human brain. Neurons having similar characteristics in an ANN are arranged in groups called layers. The neurons in a single layer are connected to those in the adjacent layers, but not to those in the same layer. The strength of connection between the two neurons in adjacent layers is represented by what is known as 'connection strength' or 'weight'. An ANN normally consists of three layers, an input layer, a hidden layer, and an output layer. An ANN can be one of the two kinds, a feed-forward type or feed-back type. The feed-forward ANNs are more popular among engineers and scientists for practical applications. In a feed-forward ANN, the information flows only in the forward direction from an input layer to the output layer. The structure of a popularly employed feed-forward ANN is shown in Figure. 1. An important step in developing an ANN model is the determination of its weights via training. There are primarily two types of training mechanisms, supervised and unsupervised. A supervised training algorithm requires an external teacher to guide the training process. The primary goal in supervised training is to minimize the error at the output layer by searching for a set of connection strengths that cause the ANN to produce outputs that are equal to or closer to the targets. The back-propagation training algorithm proposed by Rumelhar et al. (1986) is normally adopted in many engineering applications and has been used in the proposed study also.

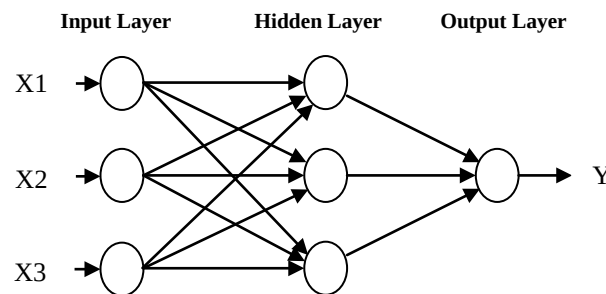


Figure. 3: Structure of a Feed Forward ANN

3.3 ANN Model Development

The development of an ANN model involves the following steps: selection of data set for training and testing of the model, identification of the input and output variables, normalization (scaling) of the data, selection of the network architecture, determining the optimum number of neurons in the hidden layer, training of the ANN models, and then at last validation of ANN model using the selected performance evaluation statistics. In this study, all the ANN models developed consisted of three layers: an input layer consisting of the neurons depending on the input data vector, a hidden layer,

and an output layer consisting of the number of neurons depending on the output vector. The input vector for different ANN models for different months were same as those for the regression models described earlier. The only neuron in the output layer represented the rainfall at time t , $P(t)$, being modelled. All the data was normalized in the range 0.1 and 0.9 in this study in order to avoid any saturation during the ANN training close to extreme values (0 and 1) of the activation function. After the input and output variables are selected, the ANN architecture of 3–N–1 was explored for capturing the complex, dynamic and non-linear monsoon rainfall for all the months in North-Central India region. That is, the input layer consisted of three neurons, output layer consisted of only one neuron ($P(t)$), and N represents the number of hidden neurons to be determined.

The unipolar sigmoid activation function was used as the transfer function at both hidden and output layers. The ANN model employed the popular Back Propagation Algorithm (BPA) using batch learning with momentum factor for its training (Rumelhart et al., 1986). The number of neurons in the hidden layer is an important step in ANN model development as it is responsible for capturing (or mapping) the dynamic and complex relationship among various input and output variables considered in developing an ANN. In this study, the optimum number of neurons in the hidden layer for each ANN model was determined using a trial and error procedure. The value of N was varied from 1 to 20 and all the 20 ANN architectures were trained using training data. The plots of N versus MSE and AIC were then prepared and the value of N giving the minimum value of MSE and/or AIC was deemed to be the best ANN architecture for a particular data set for a particular month. This procedure was repeated for all the monsoon months.

The best values of ANN model training parameters (learning rate and momentum correction factor) were determined using a trial and error procedure in which different combinations were tried before arriving at the final values. The value of learning coefficient of 0.05, momentum correction factor of 0.08, and maximum iterations of 10,000 was finally employed during training of each ANN architecture. The architecture of the ANN model (i.e. optimum number of hidden neurons) was determined by plotting MSE and AIC statistics during training v/s hidden neurons. Based on these plots, the optimum number of hidden neurons for the months of June, July, August, September, and JJAS were determined and are presented in Table 4.

Table 4: Optimum architecture of various ANN models

Months	MSE	AIC
June	7	4
July	4	4
August	5	4
September	8	3
JJAS	6	6

The various ANN models were trained using the training data sets and tested using testing data sets. Once the training of the best ANN model is completed, the trained

model structure is used to calculate various performance evaluation indices for both training and testing data sets.

4 RESULTS AND DISCUSSIONS

This section presents the results obtained in terms of various error statistics from various models developed in study. Since there are many models developed in this study for the four monsoon months, the results obtained from June month models and associated discussions are presented in this paper due to space limitations.

The result in terms of various error statistics from all the models for forecasting June rainfall are presented in Table 5. The best statistics have been presented in bold font in the table for ease in understanding. It can be observed that the performance of the naïve models is not good. The performance of all regression models is much better than the naïve models and comparable with each other. Analyzing the results during calibration/training, it can be observed that the ANN_HN_7 model performed the best in terms of E (0.8506), R (0.9261), AARE (18.69), NRMSE (0.1764), TS1 (5.98), TS50 (95.12), TS100 (100.00) while NLMRM-2 model performed the best in terms of NMBE (0.213), TS25 (82.69). Analyzing the results during testing, it can be observed that surprisingly the NLMRM-3 performed the best in terms of E (0.7227) and NLMRM-2 performed the best in terms of NMBE (0.1020), NRMSE (0.2420). However, the ANN_HN_4 performed the best in terms of R (0.7258), AARE (24.15), TS1 (8.23). Based on the various error statistics for different models during calibration/training, the ANN_HN_7 model appears to be the best and during testing ANN_HN_4 appears to be the best. Thus, any of the ANN models is suitable in modelling the June rainfall in North India.

Table 5: Error statistic from various models for June rainfall

During Calibration									
Model	E	R	AARE	NMBE	NRMSE	TS1	TS25	TS50	TS100
Persistence	-3.2542	0.2321	72.87	-77.4216	0.8838	1.92	3.85	11.54	100.00
Mean	0.0000	0.0000	53.11	0.0000	0.4285	0.00	34.62	76.92	90.38
LRM	0.7670	0.6108	21.32	0.7210	0.2140	0.00	60.78	88.24	100.00
NLRM-1	0.7751	0.6015	20.36	0.2230	1.4520	1.96	76.47	92.16	100.00
NLRM-2	0.7812	0.6078	21.12	0.2130	0.2120	5.77	82.69	92.16	98.08
NLRM-3	0.7072	0.5465	21.69	2.9500	0.2310	3.85	67.31	92.31	100.00
ANN_HN-4	0.8496	0.9257	19.10	-3.3947	0.1770	5.98	75.61	95.12	100.00
ANN_HN-7	0.8506	0.9261	18.69	-3.3833	0.1764	5.98	75.61	95.12	100.00
During Testing									
Persistence	-4.0134	0.2128	74.46	-76.0775	0.8365	0.00	0.00	3.92	100.00
Mean	0.0000	0.0000	38.75	0.0902	0.3736	1.96	45.10	76.47	94.12
LRM	0.4686	0.5526	25.39	-4.9810	0.2740	7.69	73.08	86.54	100.00
NLRM-1	0.4542	0.5163	25.31	-4.7730	0.3120	0.00	60.78	90.20	100.00
NLRM-2	0.5824	0.5689	22.92	0.1020	0.2420	1.96	72.55	90.20	100.00
NLRM-3	0.7227	0.4856	25.64	-10.5400	0.2900	0.00	58.82	88.24	100.00
ANN_HN-4	0.5184	0.7258	24.15	0.6102	0.2593	8.23	68.63	82.35	100.00
ANN_HN-7	0.5152	0.7238	24.24	0.6275	0.2601	8.23	68.63	82.35	100.00

The scatter plots during testing data sets from various models for the month of June are shown in Figure 4.4 and Figure 4.5 from regression and ANN models, respectively. It can be observed that the performance of all regression models is comparable to each other and the performance of the ANN_HN_4 model is the best. Thus, it can be said that the naïve models performed poorly, the regression model performance was only reasonable, and ANN modelling technique performed much better than the regression models in modelling the June rainfall for the NCI region in India. Thus, ANN models are recommended for forecasting June rainfall for NCI region of India.

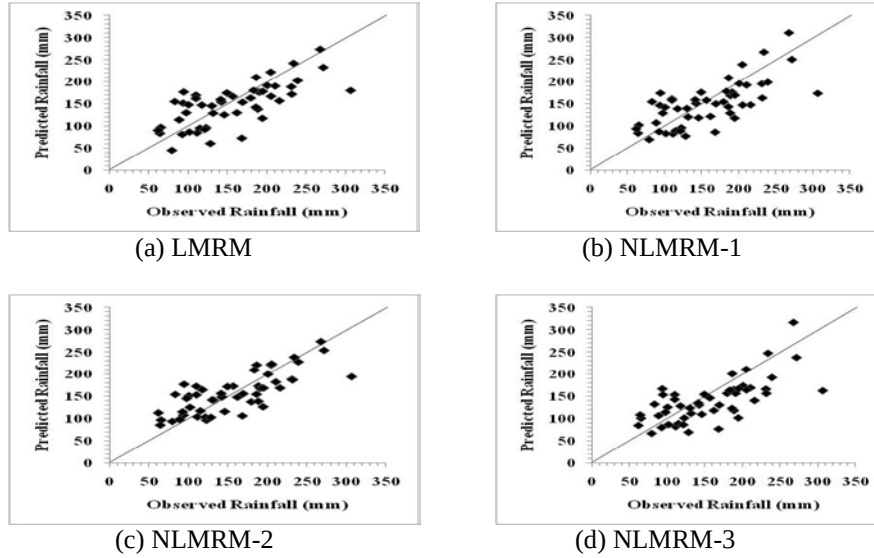


Figure 4.4: Scatter plots from regression models for June rainfall during testing

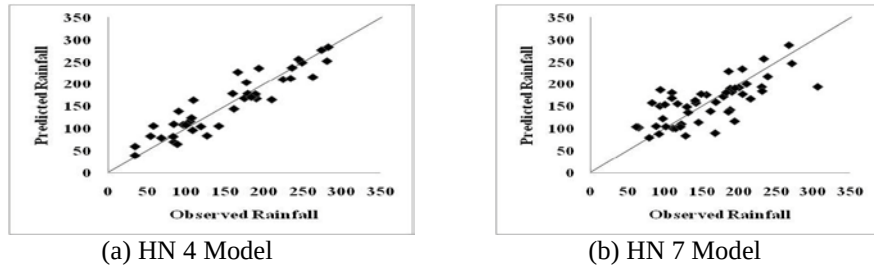


Figure 4.5: Scatter plots from ANN models for June rainfall during testing

5 CONCLUSIONS AND FUTURE SCOPE

Rainfall forecasting is important in many water resources management activities. Modeling of the complex and dynamic process of Indian monsoon has been the subject of research among the various scientists in the past. This paper presents the results of a study aimed at developing conventional and ANN models for the modeling and forecasting of the monsoon rainfall in North India. Both linear and non-linear regression models were considered among the conventional models while feed-forward type networks trained using back-propagation algorithm were considered for ANN model development. Two naive models were also considered as bench mark models.

The results obtained in this study show that the naive models are poor in nature, as expected, and hence cannot be employed in rainfall forecasting in India. The

performance of various regression models was found to be better than the naive models but can only be characterized as reasonable when it comes to applying them in important water resources management activities. Among the regression models, the non-linear regression models appeared to perform slightly better than the linear regression model, which is expected. The two ANN models developed in this study for modeling June rainfall in NCI region outperformed all other models. Among the ANN models, the one having four hidden neurons was found to be better and more compact and is recommended for forecasting June rainfall in North India. The results obtained in this study are able to demonstrate that the ANN technique is a powerful tool, which needs to be used for modeling and forecasting of the complex engineering systems.

No study is complete in itself and there is always scope for further improvements. This study considered only the rainfall and temperature data for modelling the monsoon rainfall in India. Some studies have reported that the global climatic tele-connection data also have some effect on the magnitude and patterns of monsoon rainfall in India. The ANN models considering such tele-connection variables may be able to provide a better forecasting accuracy. Further, the BPA method used for training in this study is notorious for getting trapped in local minima and global training methods e.g. genetic algorithms may provide more suitable solutions. Finally, only single hidden layer ANN models were considered in this study. Multiple hidden layer ANN models may provide a complex structure to capture the complex monsoon dynamics in an efficient manner but it remains to be investigated. It is hoped that the future research efforts will focus in some of these directions.

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