

WATER COSTS ALLOCATION IN COMPLEX SYSTEMS USING A COOPERATIVE GAME THEORY APPROACH

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ABSTRACT

The interdisciplinary nature of the planning and management of complex water resources systems requires methods that integrate the technical, economic, environmental, social and legal aspects into a comprehensive framework that allows the development of efficient and sustainable water management strategies. The Water Framework Directive 2000/60/EC (WFD) recommend that, in a river basin-scale, the pricing politics have to take into account the principle of recovery of costs and the economic sustainability of the water uses. Nevertheless the cost allocation methods currently adopted in Italy do not take into account the user's willingness to pay and, frequently, they do not permit the total costs recovering. So there is the necessity of a new approach including these requirement when defining water rates which could be applied overall in the Mediterranean Region,.

In this article we present a methodology to allocate water service costs in a water resource system among different users that try to fulfil the WFD requirements. The methodology is based on the Cooperative Game Theory (CGT) techniques and the definition of the related characteristic function using a mathematical optimization approach. The CGT provides the necessary instruments to analyze situations requiring the search of a cost sharing. The CGT approach can give solutions that should be efficient, fair and it has to provide the appropriate incentives among the involved parts. Therefore, the cost allocation in the water system has been valued as a game in which it is necessary to determine the right payoff to be assigned to the different players, which represent in our case the water users. To apply the CGT principles in a water resources system is necessary the definition of characteristic function; in this paper we used of the optimization model WARGI-Opt .

The game solutions set is represented by the so-called "core". It represents the area of the admissible cost allocation values and, through this, it is possible to supply the boundaries values of cost for every player. Inside the core there are all the allocations which satisfy the principles of equity, fairness, justice, efficiency and which guarantee the costs recovery. The core of a cooperative game can represent a useful instrument to define or to adjust water rates. Moreover it can be used as a valid support in the management of the water resources for the achievement of the economic analysis required by the WFD.

The methodology was applied to a multi reservoir and multi-demand water system in Sardinia (Italy).

1 INTRODUCTION

A central problem in planning the provision of public services when dealing with limited resources is how to determine a “fair” and “just” allocation of the management costs. This problem is particularly relevant for water systems in Europe to comply with the Water Framework Directive 2000/60/CE (EU, 2000), that promotes the necessity of an economic analysis of water use, introducing the principle of recovery of costs in order to sustain the safeguard and the qualitative and quantitative improvement of bodies of water. Nevertheless, the cost allocation methods currently adopted in Italy and in many European country, do not take into account the user's willingness to pay and, frequently, they do not permit the total costs covering. So there is the necessity of a new approach including these requirements when defining water rates. To illustrate the cost allocation methodology, application and results obtained for the Flumendosa-Campidano water system in Sardinia (Italy) are finally given.

2 COST ALLOCATION PROBLEM AND COOPERATIVE GAME THEORY

One of the basic issue of cost allocation methods is that they almost completely ignore the problem of motivation: “why should agents accept an allocation that exceeds their opportunity costs or willingness to pay?” (Young, 1985).

Cooperative Game Theory (hereafter CGT), as reported by Young (1994), provides necessary instruments to analyze situations in which it is basic to research a sharing mechanism considered efficient, just and fair by users. Furthermore, Lemaire (1984) points out that a cost allocation problem is identical to the determination of value of a cooperative game: sharing cost between users can be seen as a kind of game where it is necessary to determine the fair allocation among different players. In literature many problems and cases of cost allocation were faced using CGT principles: the approaches can be very different in the research fields and they also concern studies related to water resources (TVA, 1938; Young et al. 1982; Lippai and Heaney, 2000; Deidda et al., 2009). Certainly, one of the most important aspect of the methodology is the definition of the characteristic function of the game and in the present paper we will focus particularly on this aspect.

Preliminary to define the cooperative game for water systems, we have to explain the following definitions, extensively described by Young (1994). We will refer to $N = (1, 2, \dots, n)$ as a set of players participating in the game. Every subset $S \subseteq N$ is called “coalition”, and for $S = N$ we have the so-called “Grand Coalition” (GC). The players can represent real subjects, as the users of a water system, or member of a more abstract set as the sector of a company, or they can also represent different planning alternatives to realize commonly or separately. We refer to $c(i)$ as the “stand-alone cost”: it represents the cost connected to the user i when considered in a independent and autonomous way with respect to the other players. Instead $c(S)$ is the cost linked to a coalition S , i.e. the cost commonly sustained by all the users that belong to S . Consequently, $c(N)$ is the cost associated to the GC: i.e. the common cost sustained by all the participants of the game (all users in the water system). Finally, by convention, the cost linked to an empty coalition corresponds to zero $c(\Phi) = 0$.

The cost associated to a generic coalition (it can be formed by only one player, by a

partial coalition or by all the participants of the game GC) must represent the least cost of serving such coalition in the most efficient way: i.e. the minimum cost necessary to satisfy all players in that coalition. Moreover, the discrete function formed by costs of every coalition is called the “characteristic function” (CF) and it is a key element setting a cooperative game.

If for every pair of disjoint coalitions S' e S'' we have:

$$c(S' \cup S'') \leq c(S') + c(S'') \quad (1)$$

the CF and the related game is called sub-additive. In this case the players are stimulated to cooperate, because the unions of two group of players will determine a cost lower then the sum of the autonomous costs. A game with a sub-additive CF will be characterized by economies of scale, so the GC should be the most efficient from an economical perspective. This is the case when GC is economically more convenient to realize common projects rather than independent projects.

In order to stimulate the cooperation among the players to commonly achieve a project, it must be guaranteed the principle defined by Ransmeier (1942) as “stand alone cost test”, commonly called marginality principle. It can be referred to an individual player, eq. (2) or to a single coalition, eq. (3):

$$x_i \leq c(i) \quad \forall i \in N \quad (2)$$

$$\sum_{i \in S} x_i \leq c(S) \quad \forall S \subseteq N \quad (3)$$

where x_i is the amount of total game cost assigned to a player. According to such principle, no player or group of players, forming a coalition, would accept a cost assignment lower than the cost which he/they would sustain participating in an autonomous way, i.e. upper than its own opportunity cost.

Another principle is the so-called marginality principle or “incremental cost test” (Young, 1985). In general, the incremental cost or marginal cost of a coalition S is defined as the quantity $c(N) - c(N/S)$. According to the marginality principle, the following condition must be verified:

$$\sum_{i \in S} x_i \geq c(N) - c(N/S) \quad \forall S \subseteq N \quad (4)$$

Every player or set of players will have to sustain at least his own marginal cost when entering in a coalition. Otherwise, the coalition of pre-existent players will be in an inefficient condition to finance the entry of the new player or set of players. Rationality principle produces an incentive to the voluntary cooperation among the players, while the principle of marginality supplies conditions of equity in the game (Young, 1994).

Moreover, CGT supports solutions which include all players, so the majority of CGT solving methods are able to totally divide the GC cost among all the participant of the game (Parrachino et al., 2002). Under such hypothesis a generic solution is defined by a vector x of components $[x_1, x_2, \dots, x_n]$ such as:

$$\sum_{i \in N} x_i = c(N) \quad (5)$$

where x_i , as we said before, is the payoff assigned to player i . The eq. (5) satisfies the principle of efficiency and the principle of marginality and rationality are equivalent for this situation.

The solution of a cooperative game comes from the evaluation of CF, thus it is necessary to consider every possible coalition. Consequently, the number of players affects the complexity of the problem: given n players the number of coalitions that we have to analyze is equal to $2^n - 1$.

The game solution can be grouped in two branches (Fragnelli, 2010):

- set-theoretical solutions: which individuate a set of vectors which share the value of the game among all the players, for example the core (Gillies, 1953);
- point solutions: which individuate only one division and are more similar to the classic idea of unique solution of a problem, for example the Shapley value (Shapley, 1953) or the nucleolus (Schmeidler, 1969).

3 COST ALLOCATION METHODOLOGY

The methodology for cost allocation and the application of CGT techniques to complex water resources systems consists essentially of the following steps (Deidda et al., 2009):

- Water system analysis: functional definition of the water system and evaluation of its different aspects (hydrologic, hydraulic, infrastructural, economic, etc.).
- Cooperative game definition: identification of the players and coalitions, analysis of priorities, etc.
- CF evaluation: set up of the optimization model, calculation of the minimum associated cost for every coalition and valuation of the c.f. of the game;
- Game solution: application of CGT methods necessary to share the costs among the players.

In the first step we have to identify all the aspects for the description and characterization of water system, in particular the hydrologic, hydraulic and infrastructural ones. During this phase we have to individuate the typology of cost which characterizes the water system and that we want to share among the water users. The majority of European water systems are almost totally equipped and therefore the most important exigency is to allocate the management costs of existing infrastructures. In this case it is necessary to consider the charges relative to the ordinary and supplementary maintenance costs, to the adaptation and substitution of the works and to the energy costs for the pumping. The application example, which will be illustrated hereinafter, concerns the allocation of this typology of costs.

In the second phase of the methodology we have to identify the players and to set up the cooperative game. The players can represent both individual users and sets of them, as in case of users belonging to an unique macro-demand (for example irrigational or industrial syndicate, urban municipalities, etc...).

The third important phase of the methodology is the definition of the CF of the game. According to the definition, the CF is constituted from the set of minimum costs associated to all the possible coalitions. The necessity to value the minimum costs for

every coalition is a prerogative of the CGT. In fact, even if we wish to share the costs of the Grand Coalition, which represents the best situation for the system in sub-additive condition, it is also essential to value every coalition costs since such costs allow parameters estimation and permit a fair and efficient sharing among the players.

In the proposed development of CGT, the minimum cost of a coalition is defined as the sum of the management costs in the “minimum” set of infrastructures necessary to totally satisfy the water request of the users in that coalition. This approach has some significant differences from the methodology described in (Deidda et al., 20009). The here developed approach specifically refers to the system management and the CF is based on the optimization model WARGI (Sechi and Zuddas, 2000; Manca et al., 2004; Salis et al., 2004; Sechi and Sulis, 2009), which allows to achieve the best system performances for every coalition. The optimization of the system management is achieved by minimizing the objective function (OF) of the problem representing coalition requirements. The optimization model is easily built using the WARGI graphical interface and it is solved using Cplex (ILOG, 2007). Depending on the system infrastructures and sources and demands characterization, for each coalition it should be ranged the number and the typology of infrastructures potentially used. Consequently, the management costs of the entire system can be evaluated by a post-optimization evaluation module and it refers to the optimal flows assessment given by WARGI. Consequently, solution values are associated to that specific coalition. In this manner it is possible to evaluate the least cost (optimal system assessment) related to every coalition, and consequently to value the discrete CF of the cooperative game.

The last phase, i.e. the game solution, needs the application of the CGT methods to share the costs of the water system among the players. A subset of solutions, the so-called “core” of solutions, has been used. The core of a cooperative game is the set of allocations $x \in R^N$ such that conditions expressed from the eq. (3) and (5), (or eq. (4) and (5) in equivalent manner), were valid for every $S \subseteq N$. The core is a closed, compact, convex subset in R^N . Inside it there are several cost allocations which respect the principle of efficiency and equity and which incentive the cooperation among players. Moreover, in this way we have the chance to provide to the decision-maker an admissible range of cost to assign to each user, providing a useful and easily acceptable tool defining water rates.

3.1 Two different methodological approaches.

Developing the methodology above outlined, two different solving approaches can be considered. In a first approach, the minimum set of infrastructures in service for every coalition was defined associating the maximum priority to the users considered in the optimization phase. In such a circumstance calculating the cost of a coalition, the users belonging to it have the maximum priority in using the resources and infrastructures respect to the users out of coalition. This approach determined some problems when high levels of water requests were considered. In fact the condition of sub-additivity of the CF is not satisfied when the water demand of the system increase and the Grand Coalition is not the most efficient solution. In this case it is not possible to define the core of the game.

Consequently, a second solving approach was also used: evaluating the CF the minimum priority was assigned to the users entering in the coalition. Using this approach it is necessary to determine marginal cost of the new coalition. The marginal

cost is related to the still available resources, residual flow transfer capacities and stored water availabilities. A preliminary evaluation of these “residuals” has to be made before the player not belonging to the coalition could be admitted. Using this second approach it is then firstly necessary to run the optimization of the system considering the player out of the coalition in order to evaluate “residuals”. Operating in this way marginal costs are then evaluated. Obtained results confirm that using this second approach the CF of the game remains sub-additive, and the most efficient solution is obtained by the Grand Coalition, always being able to define the core of the cooperative game.

4 COST ALLOCATION APPLICATION

4.1 Water system description.

The methodology was applied to the Flumendosa-Campidano system in the southern part of the Sardinia island (Italy). Main resources of the water system are flows in Flumendosa, Mulargia and Fluminimannu rivers. Flows are regulated by 7 reservoirs and 7 diversion dams and delivered to demand centres through a complex supply water system made of pipelines, tunnels, canals and pumping stations. More details on the systems are given in Zucca (2011). In case of water scarcity it is necessary the exploitation of resources coming from an external system, the Tirso water system. It is possible by a more costly pumping transfer. The water system supplies civil, irrigational and industrial demands of the centre-southern zone of the island (Figure 1)

The hydrological data have been extracted from the regional programming document PSURI (RAS, 2006) that provides runoff series for 53 years of extension with a monthly time-step. According to the hydrological series, it has been set the time-length for optimization and economic evaluation.

The aim of CGT application was the evaluation of management costs sharing in the system when considering operating and maintenance costs of infrastructures and energy costs required by pumping stations. Regarding cost evaluation, we particularly utilized the PSURI (RAS, 2006) and SMGSIR (RAS, 2008) reports.

4.2 Cooperative game planning.

As regard water uses, the demand centres are grouped in three macro users: civil, irrigational and industrial. Annual total water requests are given in Table 1. In accordance with PSURI (RAS, 2006), monthly distribution is shown in Figure 2.

Each macro user represents a single player of the game, so the coalitions are formed by the single players, as [civil], [irrigational] or [industrial], by their partial aggregation, and by the Grand Coalition, [civil + irrigational + industrial].

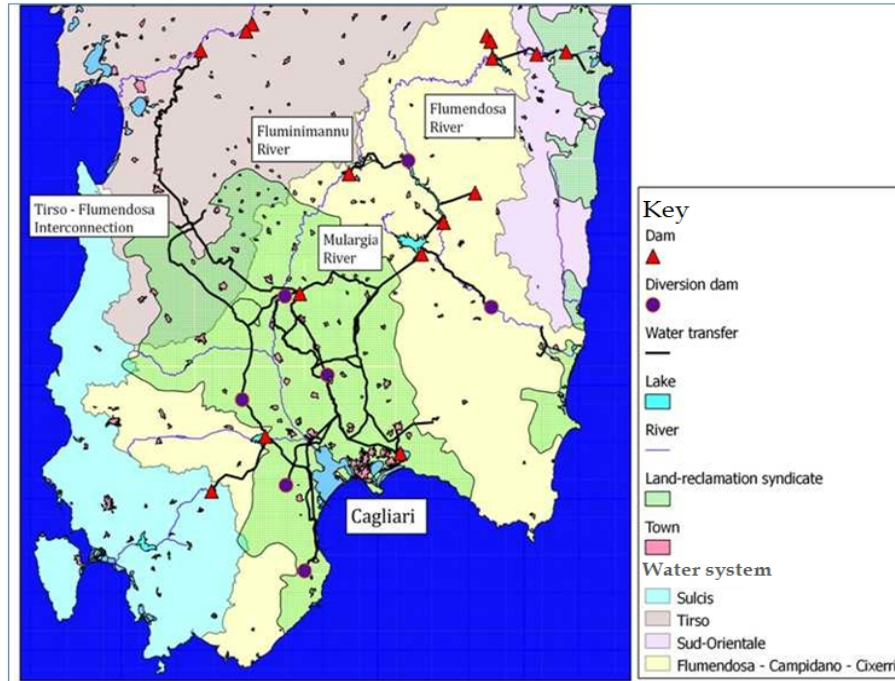


Figure 1. Flumendosa – Campidano water system

Table 1. Annual water request of the macro users.

User	Water request [Mm ³ /year]
Civil	98.70
Irrigational	81.99
Industrial	15.00
Total	195.69

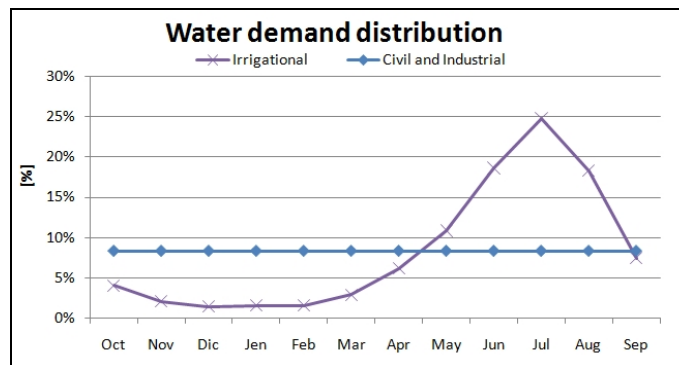


Figure 2. Percentage of water demand distribution

4.2.1 Optimization model set up

The minimum cost of every coalition is estimated by optimizing the water system management through the DSS WARGI. The objective function (OF) is evaluated for each analyzed coalition.

In the OF were considered costs regarding:

- management cost of the infrastructures, i.e. maintenance and energy cost;
- spilling cost;
- shortage cost;
- inter-periodal transfer cost for reservoirs.

The OF is constrained to:

- equation of continuity;
- maximum capacity for reservoirs nodes;
- total satisfaction of users in coalition (absence of deficit).

The spilling costs are related to penalty for the of resource which is not utilized in the system (transfers to the “sea node”). Equally, external flow transfers to the demand nodes are penalized as shortage requirements. In this way in the optimization phase it will be convenient to transfer to the sea node only the overabundant resource. A special routine in WARGI permits this will occur directly from the nodes with hydrological input excess. This is required to correctly implement the second approach, evaluating the CF: as the minimum priority was assigned to the users entering in coalition, in order to avoid the exploitation of a surplus of resource which would determine an “excess” in the use of the infrastructures and, consequently, a higher total cost.

Interperiodal transfer costs are specially linked to the resource stored during time in reservoirs. This cost is normally set to a small value in order to do not penalize the regulation capability of the reservoirs and, at the same time, to avoid the storing of unnecessary resource and to guarantee the minimum use of the infrastructures.

Inserting these typologies of costs it is possible to use the minimum amount of resource assuring, as we want, the use of the minimum set of infrastructures necessary to satisfy the water system demands.

For every coalition the optimization outputs given by WARGI provide costs and flows configuration in the system. Consequently, we can evaluate the infrastructures in service needed by each coalition and the related management cost.

For more details related to the WARGI optimization approach used evaluating the CF of the CGT see Zucca (2011).

4.3 Characteristic function evaluation.

On the basis of the minimum system management cost for every coalition, the CF values are reported in Table 2. Values refer to the whole time-length, equal to 53 years.

Then,, the Grand Coalition efficiency and comparison with other alternatives, are given in Table 3. As it was expected, from an economic point of view, the Grand Coalition is the most efficient combination for the system.

Table 2 Characteristic function of the game.

Coalition	Maintenance cost [M€]	Pumping cost [M€]	Management cost [M€] C.F.
Grand Coalition	245.87	47.49	293.36
Civil	255.13	22.65	277.78
Irrigational	190.79	29.23	220.02
Industrial	49.60	0.00	49.60
Civil - Irrigational	253.41	51.42	304.83
Irrigational - Industrial	177.31	29.23	206.54
Civil - Industrial	262.21	22.99	285.20

Table 3. Efficiency of the Grand Coalition.

Combination of Coalitions	Cost [M€]
Grand Coalition	293.36
[Industrial] + [Civil-Irrigational]	354.43
[Civil] + [Irrigational-Industrial]	484.32
[Irrigation] + [Civil-Industrial]	505.22
[Civil] – [Irrigational] + [Industrial]	547.40

4.4 Game solution.

On the basis of the evaluated CF, the game solution can be reached applying the rationality and marginality principles and boundary values are calculated (Table 4). Negative values are not admissible, therefore the minimum cost must be equal to zero.

Table 4. Boundaries cost values for players.

User	Minimum value [M€]	Maximum value [M€]
Civil	86.82	277.78
Irrigational	8.16	220.02
Industrial	0.00	49.60

As a consequence, the analytical expression of the core is reported in eq.(6). Every cost allocation that verifies (6) satisfies the principles of equity and fairness and it guarantees the total cost recovery.

Moreover, we can represent graphically the core of the game in Figure 3. In the triangle, heights are proportional to the cost of the Grand Coalition and internal points represent the set of possible (positive) cost allocations. Every side is representative of a player and the distance between one of them and one point inside the triangle gives us the allocation cost related to the player. The barycentre gives us an equalitarian sharing of costs while the vertices correspond to the situation in which total cost is completely assigned to one user. The dashed lines represent the maximum and minimum cost sustainable by every player according to Table 4 and the painted area represents the core of the game.

The obtained percentage boundary values are given in Table 5. These values are significant because they can be used by decision makers to elaborate pricing policies for the water system. Moreover, on the basis of them decision makers can also divide among the players some typologies of cost that, otherwise, cannot be easily considered.

$$\left\{ \begin{array}{l} Civil + Irrigational + Industrial = 293.36 \\ 86.82 \leq Civil \leq 277.78 \\ 8.16 \leq Irrigational \leq 220.02 \\ 0.00 \leq Industrial \leq 49.60 \end{array} \right. \quad (6)$$

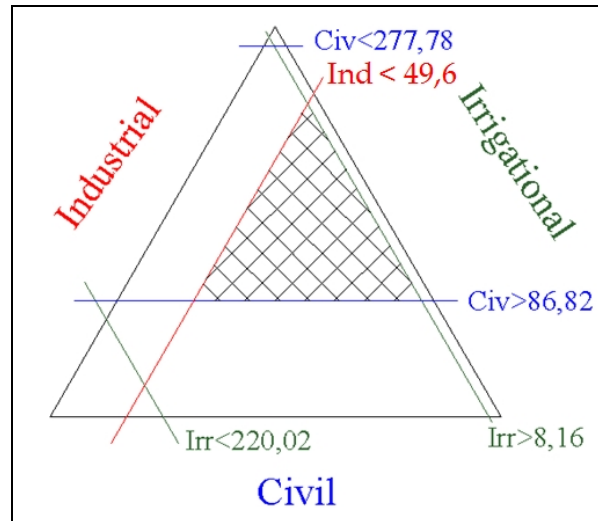


Figure 3. Core of the game

Table 5. Percentage boundaries values for the players.

User	Minimum value [%]	Maximum value [%]
Civil	29.6	94.7
Irrigational	2.8	75.0
Industrial	0.0	16.9

5 CONCLUSIONS

The problem of cost allocation for a water resources system has been faced and solved in the paper considering a CGT approach. The developed methodology permits to achieve a cost sharing policy that can be easily accepted by users; it provides an adequate justification of the adopted shearing criteria and also it promotes the cooperation among the users in order to maximize the efficiency of the management of the system.

As well known, to apply CGT, the definition of the Characteristic Function is a basic requirement. The use of the optimization graphical interface WARGI allows to correctly value the least cost for every coalition in the water system, defining the optimal set of infrastructures necessary to satisfy the water requests of the players.

The evaluation of CGT core solution represents the set of the admissible cost allocation values and it supplies the boundaries values to refer for each player. Inside the core all the allocations satisfy the principles of equity, and fairness and they guarantee the total costs recovery. Definitively, the solution core of the cooperative game can represent a useful instrument to define water rates: it still admit flexibilities frequently needed by the Water Authority. This approach can be considered as a valid tool in the management of the water resources for the achievement of the economic analysis required by the WFD. This methodology can also easily adapted to allocate other typologies of costs and it should be applicable to a wide set of water system.

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