European Water 58: 415-421, 2017.

Stochastic gradient methods for the optimization of water supply systems

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Abstract: Reductions of water deficits for users and energy savings are frequently conflicting issues when optimizing large-scale multi-reservoir and multi-user water supply systems. Undoubtedly, a high level of uncertainty due to hydrologic input variability and water demand behaviour characterizes these problems. The aim of this paper is to provide a decision support for the water system authority, in order to realize a robust decision policy, minimizing the risk of wrong future decisions. This has been done through the optimization of emergency and costly water transfer activation. Hence, a cost–risk balancing problem has been modelled to manage this problem, balancing the damages in terms of occurrences of water shortages and energy and cost requirements for emergency transfers. In Napolitano et al. (2016), we dealt with this problem using a scenario analysis approach. The obtained results were appreciable considering a limited amount of scenarios and a short time horizon. Nevertheless, if we have to increase the number of considered scenarios, taking into account the effect of climate and hydrological changes, computational problems arise. Therefore, to solve this kind of problem, it is necessary to apply a specialized approach for optimization under uncertainty. We develop herein a simulation-optimization model using the stochastic gradient methods. The study case is a multi-reservoir and multi-user water supply system in a specific area in south Sardinia (Italy), characterized by Mediterranean climate.

Key words: Stochastic gradient methods, water schedules optimization, cost–risk balancing problem

1. INTRODUCTION

The optimization of water supply systems under a high uncertainty level due to hydrologic input variability and water demand behaviour has been a subject of substantial interest in the research literature (Cunha and Sousa, 2010; Kang and Lansey, 2014; Yuan et al. 2016). In this context, pumping plants' activation schedules are a relevant topic recently treated in Napolitano et al. (2016), considering complex water resource systems management. The successful approaches to solve this kind of problems should be able to incorporate the uncertainties in the management of water systems by searching for a robust decision policy. The main aim is to provide the water system's authority with a decision support system, in order to realize a decision policy that minimizes the risk of making wrong future decisions. Through the optimization, we manage to provide the water system authority with optimal rules considering both historical and generated synthetic scenarios of hydrologic inputs to reservoirs. Using synthetic series, we analyse the climate change impacts and balance the rules while also considering future behaviour under the risk of the occurrence of shortages and the cost of early warning procedures to avoid water scarcity, mainly related to activation of emergency water transfers.

Among the considered methodologies, the scenario analysis approach (Pallottino et al., 2004; Kang and Lansey, 2014) has been proposed as an efficient optimization tool. However, it frequently needs a substantial, sometimes huge, set of hydrological scenarios to avoid the risk of system failure under a significant change of conditions over time. Despite the reasonable quality of the obtained results for standard test cases reported in Napolitano et al. (2016), this methodology encounters considerable problems in managing large-size real cases due to the high number of data and variables in scenarios. Therefore, in this study, we treat this problem with an efficient optimization model based on the stochastic gradient methods (Gaivoronski, 2005), which allows us to solve substantially larger models and to provide optimal solutions under high uncertainty.

2. OPTIMIZATION UNDER UNCERTAINTY AND THE STOCHASTIC GRADIENT METHODS

The main aim of this paper is to define the optimal thresholds for the activation of the pumping stations. When the reservoirs' storage volumes cross these thresholds, the activation of pumping is triggered. In order to optimize the activation rules, we must define a critical stored volume in reservoirs that may supply the downstream demand centres. To compose the multi-period model, we need to define the multi-period network starting from a basic graph given by a single-period static design of the water system. The multi-period network is constructed by repeating this single period model for every period using multi-period rules, as defined in Sechi and Zuddas (2008).

The climate change effects result in an increment of the different hydrological scenarios to consider their time horizon dimensions and, consequently, the level of uncertainties. In such situations, the dimensions of the multi-period model become sizable and it is very difficult to manage this stochastic problem. Therefore, to solve it efficiently we need special algorithms, designed for optimization under uncertainty. In Napolitano et al. (2016), we showed the limits of the classical scenario analysis approach (Rockafellar and Wets, 1991) caused by the high number of linear and nonlinear variables and parameters involved. We highlighted the requirements for this problem through a nonlinear variables parametrization. To solve this class of problems, the stochastic gradient methods (Birge and Louveraux, 2001; Gaivoronski, 2005) can be used as an efficient tool.

The stochastic gradient approach belongs to a class of methods specifically designed for problems with continuous distributions of random parameters and nonlinear optimization problems. These methods are suited for optimization and simulation models, where an analytical relationship between the objective function and parameters is difficult to obtain. Supply chain management, energy generation, and financial applications are some areas of research where these approaches have been successfully used (Gaivoronski, 2005).

The stochastic gradient could be described as the statistical estimate of the gradient of the objective function, which provides an optimal direction for iterative updating of the current approximation to the solution of the optimization problem. The stochastic gradient has its roots in the stochastic approximation and in the mathematical programming algorithms with gradient techniques.

These methods solve the stochastic optimization problems of the following type:

$$\text{Minimize}_{\,x\in X}\ \mathbb{E}_{\omega} f_0(x,\omega) \tag{1}$$

where $\mathbb{E}_{\omega}$ is an expected value considering $\omega \in \mathbb{R}^k$ as a vector of random parameters and $x \in X \subseteq \mathbb{R}^n$ is a vector of decision variables; X represents the set of feasible solutions.

The problem statement describes a large set of dynamic and static stochastic optimization problems. Usually, these methods can be divided into two categories: *deterministic equivalents* and *iterative sampling algorithms*.

The deterministic equivalents start by approximating the problem (1) with a problem where the original probability distribution of the random parameters ω is substituted by a discrete distribution concentrated in a finite number of values ω^i, which describe different scenarios. Therefore, the original problem could be substituted by a deterministic optimization problem with a particular structure, where the expectations in (1) are replaced by weighed sums.

Iterative sampling algorithms require statistical estimates of functions $F_i(x)$ and their gradients and Hessians. These estimates are obtained through the generation of the different observations of random parameters ω^i. The generation process supports the solution process; herein the estimates are used as substitutes for the exact values in the iterative algorithms, which are adopted from linear and nonlinear programming. Stochastic quasi-gradient (SQG) methods belong to this class of problems.

The optimization algorithm starts from an initial point x^0 and moves forward to the current approximation x^s of the optimal solution of the original problem (1) according to the following rule:

$$x^{s+1} = \pi_X(x^s - \rho_s \xi^s) \tag{2}$$

where ρ_s represents the size of the step in the direction opposite to the current estimate ξ^s of the gradient $F_0(x)$ at the point x^s .The resulting point is projected onto the set X. The projection operator π_X transforms an arbitrary point $z \in \mathbb{R}^n$ into the point $\pi_X(z) \in X$ such that:

$$\|z - \pi_X(z)\| = \min_{x \in X} \|z - x\| \tag{3}$$

the structure of the set X should allow a fast solution of the problem (3); indeed, it should be solved many times during the optimization. This means that the set X should be defined by linear constraints in order to guarantee an efficient solution process.

The crucial part of the SQG implementation consists in the evaluation of statistical estimates ξ^s of the gradient of the objective function $F_0(x) = \mathbb{E}_\omega f_0(x, \omega)$ in (1). This estimate is characterized by considerable flexibility since it is sufficient that the following condition is satisfied:

$$\mathbb{E}(\xi^s | \mathbb{B}_s) = F_{0x}(x^s) + a_s \tag{4}$$

here, $\mathbb{B}_s$ is the σ-field, defined by the history of the process, while $F_{0x}(x^s)$ represents the gradient of the function $F_0(x)$ at the point x^s. The vector ξ^s, which satisfies the property (4), is called the *stochastic gradient*.

For example, in the simplest case, the stochastic gradient ξ^s can be obtained as follows:

$$\xi^s = f_{0x}(\omega^s, x^s) \tag{5}$$

where ω^s is a single observation of a random vector ω. When the classical gradient does not exist, such as with convex but non-smooth functions, ξ^s can be estimated through a generalization of the gradient called the *stochastic quasi-gradient*.

The step-size definition is another fundamental aspect when using the SQG algorithm. Indeed, the convergence to the optimal solution depends on the selection of the correct step ρ_s, which should satisfy the following property:

$$\rho_s \geq 0, \quad \rho_s \to 0 \quad \textstyle\sum_{s=0}^{\infty} \rho_s = \infty \tag{6}$$

The required condition in (6) that the step-size tends to zero is not necessary if the precision of the stochastic gradient ξ^s increases with iterations. On the other hand, ρ_s tends to zero faster if the variance of the stochastic gradient grows with the number of iterations.

A recursive simulation and optimization process can efficiently optimize the simulation models by applying the SQG algorithm iteratively. This methodology is suited for optimization problems characterized by a large number of decision variables, described by a simulation model for each single problem. The novelty of this paper lies in the application of SQG to a real problem of water resource management that is characterized by several decision variables and an extended time horizon needed in the optimization.

3. SQG FOR A WATER RESOURCE MANAGEMENT PROBLEM

A recursive simulation and optimization algorithm based on the stochastic gradient methods has been developed for solving a water resource management problem. This is the first application of the SQG approach to such problems.

In these problems, the level of uncertainty is high, and to implement decision-making solutions it is necessary to develop correct future system management tools and to improve the security level in the decision-making process. Specifically, we can define a *cost–risk balancing process* (Gaivoronski et al., 2012a,b; Napolitano et al., 2016) considering the pumping energy and management costs (cost elements) needed in order to avoid water deficits for users (representing the risk elements). The optimization approach tries to obtain a robust decision policy, minimizing the risk of taking wrong and harmful decisions for the future. In this way, the water system's authority is able to define a water resource "target value" to deliver to the centres of consumption in order to minimize the costs and to reduce the hardships for system users. We assume that the resource in question is scarce and that, for this reason, the demand cannot be satisfied in many scenarios. In such scarcity situations, managers should develop an emergency policy to alleviate the effect of shortages. In order to do so, each user should know in advance the reduced target level of demand satisfaction that the system manager is willing to deliver to him. This target value has to be "barycentric" relative to future uncertain hydrological scenarios. In this way, we can also manage the activation of some emergency measures, such as decreasing the water distributed to the users or activating emergency transfers.

Specifically, we consider finding the optimal set of parameters q that describe the activation pumping rules while minimizing the average monthly costs, which are the sums of all costs supported in the water system management. The network state v^t (water volumes in reservoirs) evolves in discrete time $t = 1, \dots, T$ (months). At each t, water demand d^t and inflow r^t arrives. Therefore, the pumping schedules are defined by pumping rules with parameters q, and at each t; the network flows x^t are obtained by minimization of costs (7):

$$C^T(q, v^t, d^t, r^t) = \min_{x \in X} C(x, q, v^t, d^t, r^t) \tag{7}$$

subject to constraints (8) (flow continuity, bounds, etc.).

$$\Phi(x, q, v^t, d^t, r^t) = 0 \tag{8}$$

The state v^{t+1} at the beginning of period $t + 1$ is obtained from the state equation (9):

$$v^{t+1} = \Psi(x^t, q, v^t, d^t, r^t) \tag{9}$$

where the functions $C(\cdot), \Phi(\cdot)$, and $\Psi(\cdot)$ are linear with respect to (x, v).

The objective is to find the set of parameters $q = (q_1, \dots, q_n)$ that minimizes the average steady state costs, thus solving the optimization problem with an infinite time horizon (10).

$$\text{Minimize}_{q \in Q} F(Q), F(q) = \lim_{t \to \infty} \frac{1}{t} C^t(q, v^t, d^t, t^t) \tag{10}$$

where Q is some feasible set for parameters q.

This problem can be solved using SQG methods as described in the previous section. In relation to water resources, these methods have been applied to the optimization of general simulation models of discrete event dynamic systems in Dupačová et al. (1991) and Gaivoronski (2005).

The solution approach resorts to a concurrent interrelation between simulation, optimization, and evaluation steps. In the following, we give a description of the concurrent interaction between them:

– *Simulation step for all threads with LP searching for optimal flows*: the simulation process is referred to each single period t of the time horizon and is characterized by n-processes simultaneously. Each process has different sets of pumping activation threshold parameters q according to (11):

$$q^t + \delta e_k \tag{11}$$

where $\delta > 0$ is a small positive value and e_k is a vector of zeros with value 1 in the k-th position.

Here the objective function of costs referred to the single period t (7) is minimized in order to obtain an optimal configuration of the network water flows x^t.

– *Optimization step searching for stochastic gradients*: the optimization process will be applied between two consecutive periods and the new parameter configuration will be evaluated according to Equation (12):

$$q^{t+1} = \prod_Q (q^t - \rho_t \xi^t) \tag{12}$$

where $\rho_t > 0$ is the step size and $\prod_Q(\cdot)$ is the projection operator on feasible set q. The k-th component of stochastic gradient ξ^t = $(\xi_1^t, \dots, \xi_n^t)$ will be estimated by (13):

$$\xi_k^t = \frac{C_k^t - C_0^t}{\delta} \tag{13}$$

– *Evaluation step*: simultaneously, there is an estimation step based on a moving average.

$$C_0^{t+1} = (1 - \alpha_t)\bar{C}_0^t + \alpha_t C_0^t \tag{14}$$

The estimated costs C_0^{t+1} are dependent on the average costs $\bar{C}_0^t$, referred to all periods, and the costs evaluated in the previous step C_0^t. The relationship between these terms is regulated by an averaging parameter α_t.

4. CASE STUDY: SOUTHERN SARDINIA WATER SUPPLY SYSTEM

The SQG approach has been applied to the case of the water supply system of southern Sardinia (Italy), which is shown in Figure 1 considering the pumping energy needed in order to avoid the water deficits documented by different hydrological series.

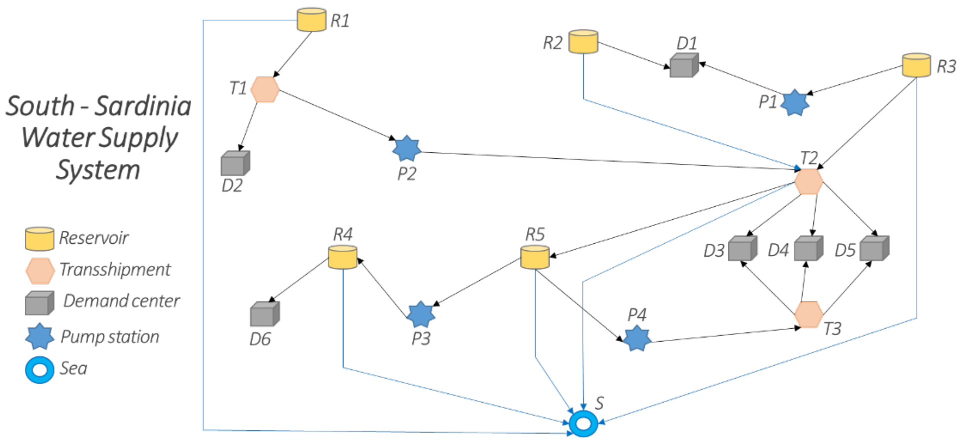

Figure 1. The south Sardinia water supply system

Water sources are given by five artificial reservoirs, while the water demands have been grouped in six centres according to three different uses: civil, irrigation, and industrial. Pump stations allow demand centres to be supplied with increased economic burden, namely by incurring pumping costs in addition to the ordinary management costs. Figure 1 simplifies the real configuration complexity of the system, but can adequately represent the system, analysing the flow configuration for values driven by the pumping stations. All the main features of this system have been extensively described in Napolitano et al. (2016). Defining the lifting rules, we assume that thresholds levels for pump activation are referred to stored volume in reservoirs that supply the downstream demand node. This could be related in some cases to a single reservoir or to two of them; in the second case the threshold value is derived from the storage volumes in both reservoirs.

4.1 SQG simulation process

In a first stage of analysis, the SQG has been implemented, adopting the activation thresholds values evaluated on historical water inputs, as described in Napolitano et al. (2016) using the scenario analysis approach; they are expressed in units of 10^6 m^3 and are equal to: S1 = 1.624, S2 = 35.451, S3 = 5.07 and S4 = 37.558.

In this paper, starting from the observed data for 53 historical years reported in the Sardinia Region Water Plan (RAS 2006), synthetic hydrological inflows to reservoirs have been evaluated by a Monte Carlo generation. In this way, the optimization time horizon has been expanded to 6360 monthly periods.

The simulation process using SQG has been developed for the synthetically generated series in order to compare the threshold values obtained in reservoirs for the activation of pumping stations. According to this, in a first stage, the thresholds are equal to the values obtained in the scenario analysis approach and running the SQG simulation these threshold values are given as parameters. The aim of this stage is to validate the pump station activation threshold results along the available database using the SQG method. SQG give us the possibility to evaluate all supported costs and the objective function value along the extended time horizon. The final objective function has been obtained as the sum of different cost elements, as shown in the left side of Table 1, under the caption “simulation process”.

The total cost function has been split into three main elements: spilling, shortage (deficit costs), and pumping. The main contributions are given by shortage and pumping costs, which have higher values.

4.2 SQG recursive simulation and optimization process

In the second phase, the SQG optimization approach has been applied to the extended time horizon, expanded to 6360 monthly periods, in order to obtain the optimized threshold values.

A relevant aspect has to be highlighted: SQG succeeds in managing this extended time horizon, which was impossible with the scenario analysis due to excessive computational requirements, as reported in Napolitano et al. (2016). The results obtained using SQG and the extended hydrology define a new set of activation thresholds for pumping transfer activation: new values are always evaluated in units of 10^6 m^3 and are equal to S1 = 0.271, S2 = 72.480, S3 = 2.775 and S4 = 21.316.

Table 1. Objective function results

Simulation Process			**Simulation and Optimization Process**		
Mean costs	10^6 € /month	10^6 € /year	Mean costs	10^6 € /month	10^6 € /year
Spilling	0.005	0.065	Spilling	0.005	0.064
Shortage	0.259	3.106	Shortage	0.231	2.767
Pumping	0.211	2.535	Pumping	0.215	2.578
Total	0.475	5.641	Total	0.451	5.345

The new values for the objective function have been obtained as shown on the right side of Table 1, under the heading “simulation and optimization process”. The total value is smaller than the result obtained in the previous stage (Table 1) due to a significant decrease of the deficit and a small increment of the pumping contributions.

5. CONCLUSIONS

The cost–risk balancing approach aims to contextually restrict deficit risks for users and to minimize the costs of managing the system in shortage conditions. The potential of the recursive simulation and optimization process based on the stochastic gradient methods is confirmed when it is applied to water resource system management, especially compared to the scenario analysis approach. Indeed, this methodology allowed us to consider substantially larger models and to provide an enhancement for the optimal solutions under large uncertainty with the need to extend the analysis to a significant number of synthetic scenarios. The results highlight an improvement in the objective function and lower computational time compared with the scenario optimization approach developed in Napolitano et al. (2016) for the same type of problem.

REFERENCES

Birge, J.R. and Louveaux, F. (2001). Introduction to Stochastic Programming. New York: Springer-Verlag.

Cunha M.C. and Sousa J.J.O. (2010). Robust design of water distribution networks for a proactive risk management. Journal of Water Resources Planning and Management, 136 (2), 227–236.

Dupačová, J., Gaivoronski, A., Kos, Z., and Szántai, T. (1991). Stochastic programming in water management: A case study and a comparison of solution techniques. European Journal of Operational Research, 52, 28–44.

Gaivoronski, A. (2005). SQG: Stochastic programming software environment. In S.W. Wallace and W.T. Ziemba, editors, Applications of Stochastic Programming. SIAM & MPS.

Gaivoronski, A., Sechi, G.M., and Zuddas, P. (2012a). Cost/risk balanced management of scarce resources using stochastic programming. European Journal of Operational Research, 216, 214–224.

Gaivoronski, A., Sechi, G.M., and Zuddas, P. (2012b). Balancing cost-risk in management optimization of water resource systems under uncertainty. Physics and Chemistry of the Earth, 42–44, 98–107.

Kang, D. and Lansey, K. (2014). Multiperiod planning of water supply infrastructure based on scenario analysis. Journal of Water Resources Planning and Management, 140 (1), 40-54.

Napolitano, J., Sechi, G. M., and Zuddas, P. (2016). Scenario optimization of pumping schedules in a complex water supply system considering a cost-risk balancing approach. Water Resource Management, 30, 5231-5246.

Pallottino, S., Sechi, G.M., & Zuddas, P. (2004). A DSS for water resources management under uncertainty by scenario analysis. Environmental Modelling & Software, 20, 1031–1042.

RAS (2006). Piano Stralcio di Bacino Regionale per l'Utilizzo delle Risorse Idriche. Regione Autonoma della Sardegna, Italy.

Rockafellar, R.T. and Wets, R.J.B. (1991). Scenarios and policy aggregation in optimization under uncertainty. Mathematics of Operation Research, 16, 119–147.

Sechi, G. M. and Zuddas, P. (2008). Multiperiod hypergraph models for water systems optimization. Water Resource Management, 22, 307–320.

Yuan, L., Zhou, J., Li, C., Xie, M., and Mo, L. (2016). Benefit and risk balance optimization for stochastic hydropower scheduling. Water Resource Management, 30, 3347–3361.