

# Reliability-based design of water distribution networks considering mechanical failures

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**Abstract:** For effective design of Water Distribution Networks (WDNs), it is necessary to consider the reliability of the system subject to mechanical and hydraulic failures. In this paper, a reliability based framework is presented for design of WDNs using self-adaptive differential evolution (SaDE) algorithm. The framework includes three main components: a simulation model, a reliability assessment model and an optimization model. Simulation model uses EPANET toolkit for simulation of flows and pressure distribution in pipe network, and its outputs are used in reliability assessment model to determine the minimum cut sets. Reliability assessment model consists of determination of system and nodal reliabilities based on minimum cut sets. The reliability is estimated in terms of demand satisfaction subject to the mechanical failure of components. The optimization model finds optimal pipe diameters considering minimization of network cost as objective subject to a minimum level of reliability and other constraints, and the model is solved by using SaDE algorithm. The application of model is illustrated by applying it to a hypothetical WDN consisting of 17 pipes, 13 nodes, a supply source and a pump. On comparison of the results of study with previous studies, it is found that the present framework yields in better solutions and saves significant amount of computational time.

**Key words:** Water Distribution Network, Reliability based design, Self-Adaptive Differential Evolution Algorithm

## 1. INTRODUCTION

Any water supply system (WSS) consists of the WDN, which is meant to receive the treated water and supply it to the consumers. WDN consists of a supply source (such as a reservoir or a tank), pipes, junctions or nodes, and other components such as valves or pumps. The problem of design of WDNs consists of determining the diameter of the pipes such that the cost is minimum subject to the minimum head requirements and conservation of mass and energy constraints. In order to ensure demand satisfaction, it is necessary to access the reliability of the network, for two types of failures: mechanical and hydraulic (Bao and Mays 1990). Mechanical failure consists of the failure or unavailability of components such as pipes, pumps, valves etc.; whereas hydraulic failure consists of changed pressure and flow conditions in the network due to uncertainty in parameters such as nodal demands and pipe roughness coefficients. Mechanical failure of any component leads to changed flow and pressure conditions inside the network, which may lead to reduced supply at certain nodes. The level of demand satisfaction in such cases is represented as the reliability of the WDN.

Mechanical failure of the pipes can occur due to various factors such as excessive internal pressure, corrosion, sudden failure of pump, external loadings etc. Various factors can be attributed to the rate of failure of pipes such as pipe diameter, age, length, material and previous breakage data. Pipe age and diameter have been found to be the most influential parameters affecting future failure rates of pipes (Kettler and Goulter 1985). Various models have been developed for assessing the future failure rates of the components, such as neural networks (Achim *et al.* 2007), Bayesian Belief Network (Kabir *et al.* 2015). However, the relationships developed will vary with the study area and hence requires a detailed analysis based on previous breakage records (Asnaashari *et al.* 2013). The developed relationship will be useful in assessing the mechanical reliability of the network.

Reliability has been assessed in various ways in the past. Shamir and Howard (1981), Su *et al.* (1987) and Fujiwara and Silva (1990) measured reliability as the shortfall due to failure of links. Bao and Mays (1990) considered the uncertainties in demand and roughness coefficients for estimating the reliability of the system. Shibu and Reddy (2014) considered fuzzy random demands for assessment of the reliability of the WDNs. Reliability has been found to be an important consideration for the design of WDNs.

Initially the problem of design of WDNs was considered as a single objective problem considering only minimization of cost. In recent past, the WDN problems were solved using different meta-heuristic methods such as Genetic Algorithm (GA; Kadu *et al.* 2008), Ant Colony Optimization (ACO; Maier *et al.* 2003), Particle swarm optimization (PSO; Montalvo *et al.* 2008), Differential Evolution (DE, Suribabu 2010) etc. Also, in few studies, the concept of reliability was introduced and reliability based design was performed using single objective as well as multi objective techniques (Fu and Kapelan, 2011; Shibu and Reddy, 2014).

In the field of optimal design of WDNs, the use of Evolutionary Algorithms (EAs) has increased tremendously due to the various inherent advantages they possess, such as fast convergence, ability to avoid local optima, obtaining a set of non-dominated solutions etc. Two EAs namely GA and DE are the most widely used algorithms in the field of optimal design of WDNs. It has been seen that DE converges faster than GA (Suribabu 2010). However, use of DE requires sensitivity analysis in order to determine the optimal values of mutation and cross over factors. In order to overcome this difficulty, many self-adaptive DE algorithms have been developed, such as fuzzy adaptive DE (FADE; Liu and Lampinen 2005), Self-Adaptive DE (SaDE; Qin and Suganthan 2005), DE with Self-Adaptive Population (DESAP; Teo 2006), jDE (Brest *et al.* 2006) and Adaptive DE with optional external archive (JADE; Zhang and Sanderson 2009). Out of these algorithms, SaDE is the first self-adaptive algorithm, which has been found to perform better compared to the previously existing algorithms, in terms of obtaining optimal solutions and convergence speed (Zhang and Sanderson 2009). The present study thus implements SaDE for the purpose of WDN design problem.

The objective of the present study is to perform reliability based design of WDNs using self-adaptive DE algorithm, through application to a hypothetical WDN and evaluate its performance by comparing with previous studies.

## 2. METHODOLOGY

The overall methodology consists of three components: a simulation model, a reliability assessment model and an optimization model. The simulations have been performed by using EPANET 2 toolkit. Reliability has been estimated using minimum cut set method (Su *et al.* 1987). Optimization has been performed using SaDE algorithm (Qin and Suganthan 2005).

### 2.1 Problem formulation

The problem is formulated as an optimization problem for determining the size of the pipes, such that the cost would be minimum with a minimum level of reliability and following the constraints of minimum head requirement and conservation of mass and energy. Mathematically the problem can be stated as

$$\text{Minimize Cost} = \sum_{i=1}^{n_p} f(D_i)L_i \quad (1)$$

subject to:

$$H \geq H_{min} \quad (\text{for all nodes}) \quad (2)$$

$$\sum HL_i - \sum E_p = 0 \quad (\text{for all loops}) \quad (3)$$

$$\sum Q_{in} - \sum Q_{out} = 0 \quad (\text{for all nodes}) \quad (4)$$

$$R \geq R_{min} \quad (\text{for both system and individual demand nodes}) \quad (5)$$

where  $HL_i = \frac{10.68 Q^{1.85} L_i}{C_{HW}^{1.85} D_i^{4.87}}$  and  $Q = \frac{\pi}{4} D^2 V$

Here,  $D_i$ ,  $L_i$  = diameter and length of pipe  $i$ ,  $H$  is the head,  $H_{min}$  is the minimum required head at any node,  $HL$  is the head loss for a pipe,  $E_p$  is the energy added by the pump,  $R$  is the reliability and  $R_{min}$  is the minimum required reliability value,  $Q_{in}$  and  $Q_{out}$  are the discharges flowing towards and away from a node respectively,  $C_{HW}$  is the Hazen William's roughness coefficient and  $V$  is the velocity of flow for any particular pipe.

## 2.2 Simulation model

Simulations have been performed by using EPANET 2 toolkit, in order to generate the pressure and flow conditions in the network. EPANET is a hydraulic simulation tool useful for design of water piping system and it was developed in 1993 by the United States Environmental Protection Agency (EPA) Water Supply and Water Resources Division (Rossman 2000). It is a computer program that performs extended period simulation of hydraulic and water quality behavior within pressurized pipe networks.

EPANET simulates the water distribution network using the Gradient Algorithm, and the solutions of flows and heads at a particular time step is obtained by solving a set of equations which mainly involves: (a) Conservation of mass equation at a junction (b) Head loss equation for a loop (Todini and Pilati 1987). In this study, the pipe failure conditions have been simulated by setting the status of the pipe as close, using the EPANET DLL files, through matlab coding.

## 2.3 Reliability assessment model

Reliability is estimated using the minimum cut set method. A minimum cut set consists of the minimum number of components which when fail together causes system failure, however if any of the components is in working state, then the system failure does not occur. In order to calculate the reliability of the network using this method, first the minimum cut sets are found, which can be done by simulating failure of different pipes (i) individually (ii) combinations of pipes which do not form minimum cut sets for single pipe failure case. The probability of failure of each cut set is calculated as multiplication of failure probabilities of all the members forming that cut set.

Thus, if there are  $k$  components in a minimum cut set, then the failure probability of that cut set can be calculated as

$$P(MC_i) = \prod_{j=1}^k P_j = P_1 \times P_2 \times \dots \times P_k \quad (6)$$

If there are  $m$  minimum cut sets, then the failure probability of the system will be equal to the sum of failure probabilities of all the cut sets, i.e.

$$P_s = \sum_{i=1}^m P(MC_i) = P(MC_1) + P(MC_2) + \dots + P(MC_m) \quad (7)$$

Reliability can be calculated as

$$R = 1 - P_s \quad (8)$$

#### 2.4 SaDE algorithm

Optimization is performed by using Self Adaptive Differential Evolution (SaDE) algorithm (Qin and Suganthan, 2005). The basic steps involved in SaDE are as follows:

1. Initialise the population with some random values.
2. Set initial values of the means of mutation and cross over factors as 0.5 each and standard deviation as 0.3 and 0.1 respectively.
3. Generate initial random values of mutation and cross over factors for each individual from a normal distribution with the specified mean and standard deviation values.
4. Now calculate the cost and reliability values for each individual.
5. Perform mutation for each individual using the relation

$$V_{i,G} = X_{r_1,G} + F_i(X_{r_2,G} - X_{r_3,G}) \quad (9)$$

where  $V_{i,G}$  = mutant vector associated with population  $i$  and generation  $G$

$X_{r_1,G}$ ,  $X_{r_2,G}$  and  $X_{r_3,G}$  are three randomly chosen population vectors from the present generation and  $F_i$  is the mutation factor for population vector  $i$ .

6. The next step is to perform cross over, which is nothing but to select certain features from the original vector and rest from the mutant vector, to generate a trial vector. Cross over can be performed using the equation

$$u_{j,i,G} = \begin{cases} v_{j,i,G} & \text{if } (\text{rand}_j[0,1] \leq CR \text{ or } (j = j_{rand})) \\ x_{j,i,G} & \text{otherwise} \end{cases} \quad (10)$$

where,  $u_{j,i,G}$  is the trial vector,  $\text{rand}_j$  is a random value between 0 to 1 for each dimension of the vector and  $j_{rand}$  is a random number from 1 to  $D$ , the dimension of each vector.

7. Now, for the trial vector generated, compute the cost and reliability values. The fitness function can be calculated as cost plus penalty, where penalty is computed using the rule

$$\text{Penalty} = \begin{cases} (R_{\min} - R) \times 10^6 & \text{if } R < R_{\min} \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

8. Generate the new population by applying the selection process. Here, the selection is kept simple by comparing the fitness function of the trial vector with the corresponding original population vector. The one with better fitness function is forwarded to the next generation.
9. The process is continued till the convergence criteria is fulfilled. Here convergence criteria is that either a maximum number of iterations are achieved or there is no improvement in the fitness function value for a certain number of iterations (say 20).

In order to update the values of mutation and cross over factors, the values of these factors leading to successful trial vectors are noted for a certain number of iterations, say 10. After the required number of iterations are over, the mean of mutation and crossover factors are updated as equal to the mean of the successful values of these factors respectively. New set of mutation and crossover factors are then generated from normal distributions with the updated mean values.

The overall working procedure for the model has been depicted in Figure 1.

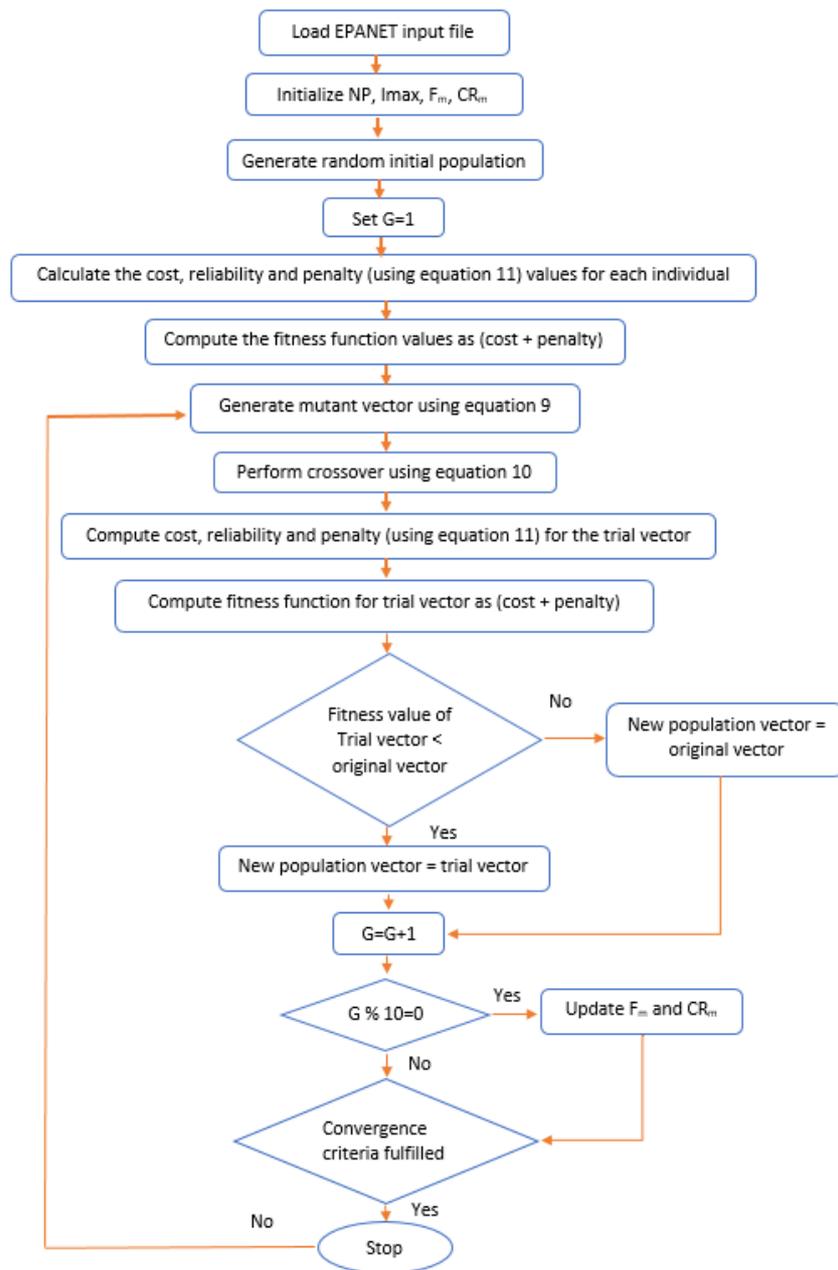


Figure 1. Overall Model Framework

### 3. RESULTS AND DISCUSSION

The model is applied on a hypothetical WDN (Su *et al.* 1987), as shown in Figure 2. Complete data set for the network can be found in Su *et al.* (1987).

The probability of failure of pipes was modelled using Poisson's distribution as follows:

$$P_j = 1 - e^{-\beta_j} \quad (11)$$

where  $P_j$  is the probability of failure of  $j^{\text{th}}$  pipe and  $\beta_j$  is the expected number of failures per year for pipe  $j$  and is given by

$$\beta_j = r_j L_j \quad (12)$$

where  $r_j$  = expected number of failures per year per unit length of pipe  $j$  and can be calculated as

$$r_j = \frac{0.6858}{D_j^{3.26}} + \frac{2.7158}{D_j^{1.3131}} + \frac{2.7685}{D_j^{3.5792}} + 0.042 \quad (13)$$

The minimum required head at all the nodes for full supply is taken as 28 m. The minimum system reliability is set as 0.8, whereas the minimum nodal reliability is taken as 0.95 for all the nodes.

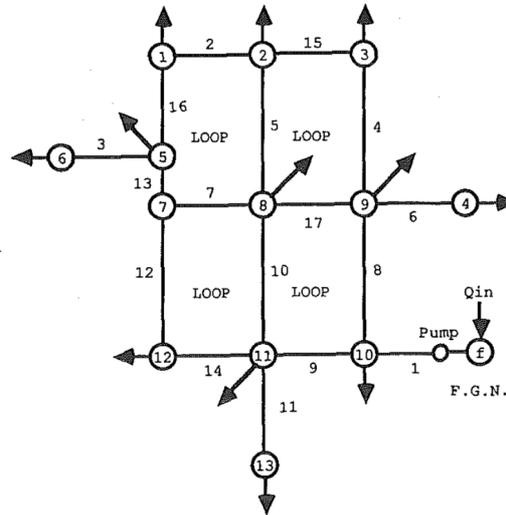


Figure 2. Hypothetical Network (Su et al. 1987)

The model is run for different population sizes for 10 trial runs each. It was found that for population sizes of 100, 150, 200 and 250, the average minimum cost was found as  $4.3125 \times 10^5$ ,  $3.4638 \times 10^5$ ,  $3.4384 \times 10^5$  and  $4.1733 \times 10^5$  \$ respectively, and requiring minimum number of function evaluations for obtaining optimal solution as 28800, 32100, 31500 and 48750 respectively. Thus, population size of 200 performs best in terms of average minimum cost and number of function evaluations needed to obtain the optimal solution. The corresponding  $F_m$  and  $CR_m$  values are obtained as 0.3 and 0.9 respectively. The algorithm converges to optimal solution at around 150 iterations.

Table 1. Comparison of optimal solutions obtained in the present study with previous study of Su et al. (1987).

Pipe id	Optimal diameter value (inches)	
	Present Study	Su et al. (1987)
1	20	34.55
2	6	14.93
3	12	16.96
4	6	9.71
5	6	15.83
6	10	16.98
7	12	26.38
8	20	21.53
9	20	19.01
10	12	6.94
11	12	13.63
12	12	22.37
13	12	16.20
14	12	5.13
15	6	11.12
16	6	11.53
17	15	9.30
Cost (\$)	$3.4204 \times 10^5$	$6.1626 \times 10^5$
Reliability	0.8463	0.83797

The obtained optimal values of pipe diameters are listed in Table 1. The optimal cost is obtained as  $3.4204 \times 10^5$  \$ with a reliability value of 0.8463. The average computational time needed is 70 minutes. The results as obtained by Su *et al.* (1987), led to a cost of  $6.1626 \times 10^5$  \$ with reliability of 0.8379 requiring 200.5 minutes of computational time. Thus, the SaDE algorithm is able to find better solution in terms of optimality and is efficient in terms of computational time needed. Also, in the algorithm used by Su *et al.* (1987), the diameter values have been assumed as continuous, which is not the case in real WDNs. This drawback has been eliminated in the present study, by incorporating discretization of the decision variables to the practically available pipe sizes.

#### 4. CONCLUSIONS

This paper presented self-adaptive differential evolution (SaDE) algorithm for reliability based design of WDNs considering mechanical failure of pipes. The methodology can also be used in optimizing the extension of an existing WDN, by keeping the diameters of existing pipes as constant values and optimizing the diameters of extensions. The model is found to be efficient in terms of the obtained optimal solution. The solutions obtained are more accurate and practically feasible since it considers discrete value for the decision variables. Moreover, the solutions can be obtained in reasonable amount of computational time. The self-adaptive nature of the algorithm eliminates the need of initializing the mutation and crossover factors, which is basically problem dependent and requires proper sensitivity analysis to determine the optimal values of these parameters. The need for such analysis is eliminated in the present study. The algorithm can be suitably modified to incorporate other factors such as failure of pumps, location of valves and hydraulic failures.

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