

## Modeling and optimization control of a hybrid water system

H. Mu<sup>1\*</sup>, T. Rauschenbach<sup>2</sup>, D. Karamanzia<sup>2</sup> and N. Cai<sup>1</sup>

<sup>1</sup> Fraunhofer Beijing Representative Office, Landmark Tower II 0611, Chaoyang, Beijing, PR. China

<sup>2</sup> Advanced System Technology (AST) Branch of Fraunhofer IOSB, Am Vogelherd 50, 98693 Ilmenau, Germany

\* e-mail: muh@fraunhofer.com.cn

**Abstract:** In most deltas, rivers and channels compose to a water network system. Different water conservancy projects, such as pumping stations, sluices, and barriers are constructed in the water system. These water conservancy projects change the natural water flows, i.e. the direction of a water flow is no longer controlled by the natural terrain, but by the difference in the state of the water heads between sluice gates. Even the flow of the rivers is not clear anymore. The water system influenced by a discrete state control has evolved into a complex hybrid water system. The conventional way to manage such a water system is to monitor water level changes and to control different sluice gates based on experience. However, such management is challenging and causes a serious loss of energy and resources. Therefore, a model-based optimal scheduling process is expected in practice. Unfortunately traditional hydrological models are unable to model hybrid water systems totally. Interdisciplinary research is needed, especially inheriting some approaches from the field of automation technology. This paper models a complex hybrid water system with a scheduling optimization approach. The definition “virtual tank” is introduced to solve the bi-directional flow problem of the hybrid water system. Scheduling processes in short or midterm periods are optimized with the Model Predictive Control Method (MPC). The modeling and optimization approaches are demonstrated in a case study – Smart flood control in the hybrid water network area of Jiangyin, PR China.

**Key words:** Bi-directional flow, Hybrid water system, MPC-MLD optimisation, Water network modelling

### 1. INTRODUCTION

Artificial water conservancy projects, such as pumping stations, sluices, and barriers, densely cover a plain river network area. They change the natural hydrologic dynamic flow and form a hybrid water system (Definition of Hybrid Water System see chapter 2). The water flow in the river is no longer caused by the natural terrain, but by the difference in the water level of sluices on the ends of the river. Changing the state of a sluice causes a change in the water flow situation. Therefore, for the management of water resource scheduling, not only the dynamic change of the continuous variable should be considered, but also the discrete event caused by opening and closing the sluices should be analyzed. In order to solve such complex hybrid water systems, two concepts are developed and implemented in this work:

1. With the introduction of an innovated concept, the “virtual tank”, the modeling process for the dynamic flow changes based on state changes (sluice gate open or closed) in hybrid water systems can be solved. The modeling not only provides a visual simulation, which makes it easy for decision makers to understand the water situation, but also provides a basement for optimal scheduling.
2. Model Predictive Control (MPC) Method based on Mixed Logical Dynamic (MLD), is applied to solve the optimization scheduling in short and midterm periods. The system can calculate and deliver suggestions for control strategies in time, based on the weather forecast and scenario definition.

The goal of this study is to set up scenarios with short-term regional hydrological changes, i.e. heavy precipitation in a short timeframe. The hybrid water system is then modelled to forecast the water situation. According to the limiting conditions of engineering projects (dams, gates, sluices,

barriers) and optimization objectives for scheduling, the management strategies are then determined by the MPC-MLD approach. This paper refers to some research results of Delft University of Technology in Netherlands (van Ekeren, 2011), but the models have been redefined in this work with the innovated concept called the “virtual tank”, and was implemented as a large scale research in a dense river network area of Yangzi delta in Jiangyin (987.5 km<sup>2</sup>), Jiangsu Province, PR China.

## 2. METHODOLOGY

### 2.1 Modelling

First of all, we must define the term “hybrid system”. The term “hybrid system” originates from Cybernetic science. In this system, not only do dynamic continuous variables exist, but also discrete state events. A hybrid water system is a special reflection in water resources management. Figure 1 shows a typical hybrid water system with natural river flow, sluices, etc.

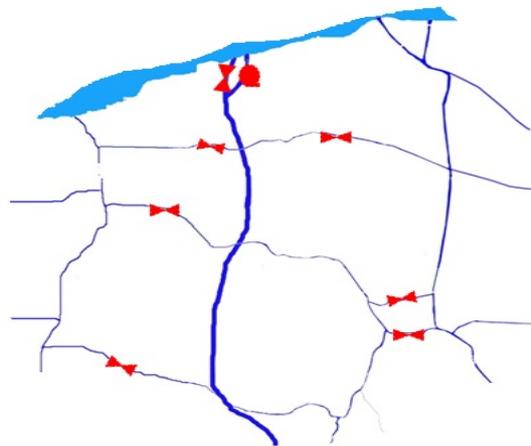


Figure 1. A typical hybrid water system

In Figure 1, only the big river in the north is still flowing relatively natural in the entire river system. Sluice gates are densely constructed on other rivers. Due to the opening and closing of these sluice gates, the water level, the flow activity, and the direction of water are changed actively. Therefore, in order to simulate the actual interactions of the hydrological dynamic variables, i.e. the river flow, and the discrete state variables caused by opening and closing pump stations and sluice gates, this paper introduces an innovated concept of the virtual tank. A virtual tank can be applied at two different application levels:

1. The intersection of two or more channels is assumed to be a virtual tank. Neighboring virtual tanks are connected by channel between them. For the flow calculation the Chézy model is introduced as follows:

$$f_{chézy}(x_i(k), x_j(k)) = A_{c,ij}(x_i(k), x_j(k)) C_{ij} \text{Sign}(x_j - x_i) \times \sqrt{\frac{R_{ij}(x_i(k), x_j(k)) |x_j - x_i|}{l_{ij}}} \quad (1)$$

$x_i$  and  $x_j$  are the water level of two virtual tanks  $i$  and  $j$ ,  $A_{c,ij}(x_i(k), x_j(k))$  is the minimum section area of flow  $q_{ij}(x_i(k), x_j(k))$ ,  $C_{ij}$  is roughness coefficient,  $R_{ij}(x_i(k), x_j(k))$  is hydraulic radius of flow  $q_{ij}(x_i(k), x_j(k))$ ,  $l_{ij}$  is the length of the connection between  $i$  and  $j$ .  $\text{Sign}(x_j - x_i)$  express the flow direction.

The model calculates the flow direction in each time interval. When the calculated value is positive the flow is the same as the initial direction of flow; when it is negative the flow is in the opposite direction. Figure 2 shows the structure of the first concept of the virtual tank.

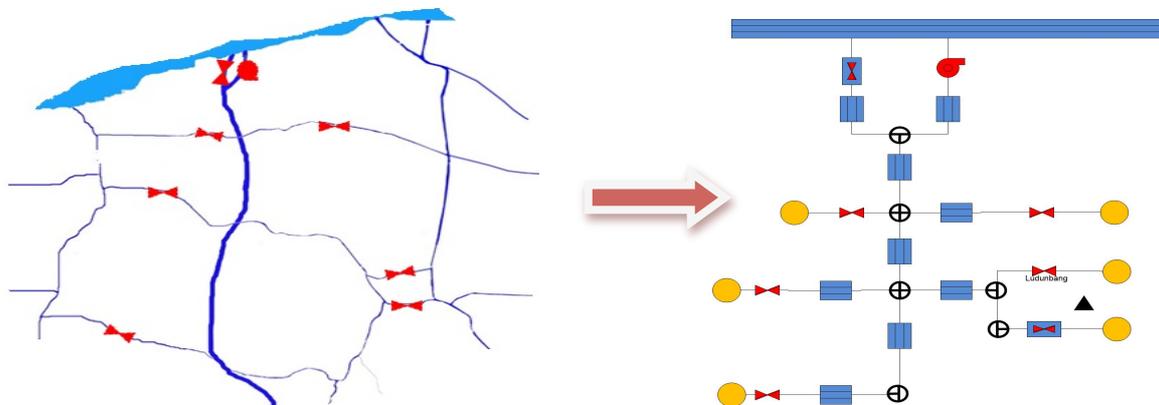


Figure 2. Structure of the first description of the model based on the virtual tank

This application is good for modeling of river networks considering the time discrete state changes. This enables a clear expression of the water situation. It can be used for testing the control strategies, river flood control, and water diversion management.

2. The second application of the virtual tank concept is considering the catchment area, the administrative area, and the river channel characteristics. The study area is divided into a set of submodel areas, each partition is assumed to be a virtual tank. Both neighboring partitions, i.e. virtual tanks, are connected by a main channel in between. Figure 3 shows an expression of the second application of the concept of a virtual tank:

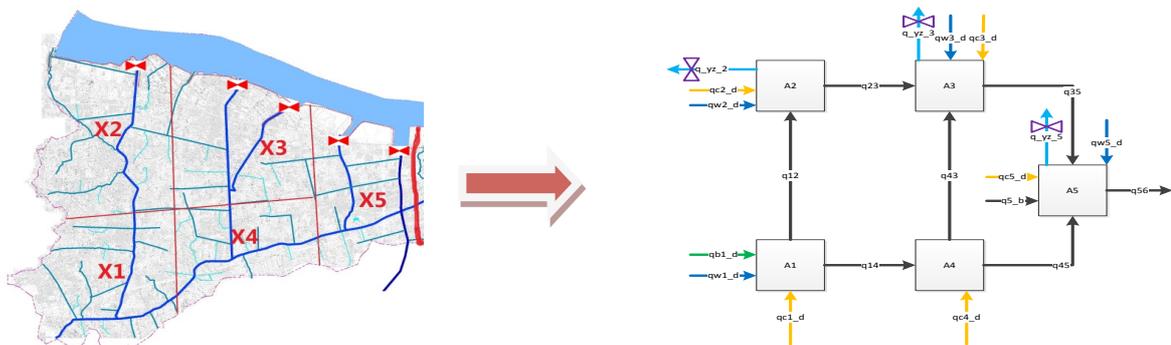


Figure 3. Higher level of the application of a virtual tank

This application is suitable for modelling of large regions. The region is divided into different sub-regions (virtual tanks). All tanks were modelled and their behaviours were analysed. This method can express the impact on each model area caused by the rainfall and runoff. It has a profound significance for flood control in the whole region.

Based on the concept of the virtual tank, a hybrid water system can be modeled. Take Figure 3 for example. This water system consists of 3 sluices and 5 virtual tanks, which are connected by main channels in between. Some additional inflows to the virtual tanks are also considered respectively. The state of each tank is represented by the water level. According to the mass balance principle, the change of the water level in the discretized time step  $k+1$  is described as follows:

Virtual Tank A1:

$$X_1(k+1) = \frac{T_s}{A_{s1}} [q_{b1\_d}(k) + q_{w1\_d}(k) + q_{c1\_d}(k) - q_{12}(X_1(k), X_2(k))] + X_1(k) \quad (2)$$

Virtual Tank A2:

$$X_2(k+1) = \frac{T_s}{A_{s2}} [-q_{yz\_2}(X_2(k), Yz\_L_2(k), u_2(k)) + q_{w2\_d}(k) + q_{c2\_d}(k) + q_{12}(X_1(k), X_2(k)) - q_{23}(X_2(k), X_3(k))] + X_2(k) \quad (3)$$

The Virtual Tank A3 to A5 expressed in the same way, where  $k$  is the discrete time step,  $T_s$  is the sample time;  $A_{si}$  ( $i=1,2,\dots,5$ ) is the surface area of the water surface in each tank;  $X_i(k)$  ( $i=1,2,\dots,5$ ) is the water level of each corresponding virtual tank at  $k$  time step;  $q_{ci\_d}(k)$  ( $i=1,2,3,4,5$ ) is the disturbance inflow due to the precipitation and transpiration;  $q_{w1\_d}(k)$ ,  $q_{w2\_d}(k)$ ,  $q_{w3\_d}(k)$ ,  $q_{w5\_d}(k)$  are the inflows due to the water supply from the external river;  $q_{b1\_d}(k)$ ,  $q_{5\_b}(k)$  are the disturbance inflows from the boundaries;  $q_{yz\_2}(X_2(k), Yz\_L_2(k), u_2(k))$ ,  $q_{yz\_3}(X_3(k), Yz\_L_3(k), u_3(k))$ ,  $q_{yz\_5}(X_5(k), Yz\_L_5(k), u_5(k))$  are flows between the large north river and the local channels, which controlled by the sluices; and  $q_{12}(X_1(k), X_2(k))$ ,  $q_{23}(X_2(k), X_3(k))$ ,  $q_{43}(X_4(k), X_3(k))$ ,  $q_{35}(X_3(k), X_5(k))$ ,  $q_{14}(X_1(k), X_4(k))$ ,  $q_{45}(X_4(k), X_5(k))$  and  $q_{56}(X_5(k), X_6(k))$  are the flows between two connected tanks, this is equivalent to  $q_{ij}(X_i(k), X_j(k))$  in the Chézy formula (van Ekeren, 2010), and can be expressed as:

$$q_{ij}(x_i(k), x_j(k)) = f_{chézy}(x_i(k), x_j(k)) \quad (4)$$

$$f_{chézy}(x_i(k), x_j(k)) = A_{c,ij}(x_i(k), x_j(k)) C_{ij} \text{Sign}(x_j - x_i) \times \sqrt{\frac{R_{ij}(x_i(k), x_j(k)) |x_j - x_i|}{l_{ij}}} \quad (5)$$

The section area variable  $A_{c,ij}(x_i(k), x_j(k))$  and the hydraulic radius  $R_{ij}(x_i(k), x_j(k))$  are the functions of the water level. Their values are inflected according to the change in water level between the two tanks in each time step. The section area variable  $h_{mean}$  is approximated with the average value of  $x_i, x_j$ . Under this kind of approximation, the area of the cross section and the hydraulic radius are only related to the channel parameters ( $b$  the width of the channel and  $m$  the slope):

$$A_{c,ij}(x_i(k), x_j(k)) = (b + m \times h_{mean}) \times h_{mean} \quad (6)$$

wetted perimeter calculation formula:

$$P_{c,ij}(x_i(k), x_j(k)) = b + 2 \times h_{mean} \sqrt{1 + m^2} \quad (7)$$

It follows that the hydraulic radius can be calculated as:

$$R_{ij}(x_i(k), x_j(k)) = \frac{A_{c,ij}(x_i(k), x_j(k))}{P_{c,ij}(x_i(k), x_j(k))} = \frac{A_{c,ij}(x_i(k), x_j(k))}{b + 2 \times h_{mean} \sqrt{1 + m^2}} \quad (8)$$

The flows  $q_{yz\_2}(X_2(k), Yz\_L_2(k), u_2(k))$ ,  $q_{yz\_3}(X_3(k), Yz\_L_3(k), u_3(k))$ ,  $q_{yz\_5}(X_5(k), Yz\_L_5(k), u_5(k))$  are described by the formula of Chézy, the difference here is that a key variable  $u_i(k)$  is added, which represents the state of corresponding sluices, therefore the formula can be rewritten as:

$$q_{yz\_i}(X_i(k), Yz\_L_i(k), u_i(k)) = u_i(k) f_{chézy}(x_i(k), Yz\_L_i(k)) \quad (9)$$

where,  $Y_{Z\_L_i}(k)$  is the water level of the north river (see Figure 2), and the state of the sluice is given by:

$$u_i(k) = \begin{cases} 0 & \text{if the sluice closed at the time } k \\ 1 & \text{otherwise} \end{cases} \quad (10)$$

Now, all the variables are defined. Through the time series of the input data (the disturbance inflows) and static state data, the water level output data can be calculated under the conditions of the initial data. Then virtual tank model parameters ( $A_{c,ij}(x_i(k), x_j(k)), l_{ij}$ ) can be inferred backwards by comparing the actual levels of output data.

## 2.2 Optimization using MPC based on MLD

MPC (Model Predictive Control) originates from the process industry, but nowadays MPC is used in many different fields. In water resources management, especially in dealing with water hybrid systems, MPC is quite new. MPC as a technology of Advanced Process Control (APC) is based on a prediction model. Several models could be selected for the hybrid system, such as Hybrid Automata, Petri Net, Piece Wise Affine, and MLD (mixed logical dynamical). Because the virtual tank model can be easily extended to the MLD model, the MLD model is selected here. (Zhang, 2007).

An MLD system has these well-known expressions:

$$X(k+1) = A(k)X(k) + B_1(k)U(k) + B_2(k)\delta(k) + B_3(k)Z(k) + B_4(k)D(k) \quad (11)$$

$$Y(k) = C(k)X(k) + D_1(k)U(k) + D_2(k)\delta(k) + D_3(k)Z(k) + D_4(k)D(k) \quad (12)$$

$$E_1(k)X(k) + E_2(k)U(k) + E_3(k)\delta(k) + E_4(k)Z(k) + E_5(k)D(k) \leq G \quad (13)$$

The expression shows a MLD system which is defined as a linear relation consisting of continuous variables and binary variables, where,  $X$  is the state of the system,  $Y$  is the output variable of the system,  $U$  is defined as input vector and is a control variable of the system,  $Y$  and  $U$  have the same structure like  $X$ .  $\delta$  is the logical statement of the system:  $\delta \in \{0,1\}^v$ ;  $Z$  is the auxiliary continuous statement:  $z(k) = x(k) * \delta(k)$ ,  $z(k) \in \mathfrak{R}^c$ ;  $D$  represents disturbance variables.  $z(k)$  Equation is nonlinear, therefore some inequality constraints have to be defined (van Ekeren, 2010):

$$z(k) \leq M\delta(k) \quad (14)$$

$$z(k) \geq m\delta(k) \quad (15)$$

$$z(k) \leq x(k) - m[1 - \delta(k)] \quad (16)$$

$$z(k) \geq x(k) - M[1 - \delta(k)] \quad (17)$$

In the case study,  $M$  is defined as the maximum possible value of the state of virtual tank, and  $m$  is the minimum possible value of the virtual tank. The relation (2), (3) are multiple expressions, and the part of the nonlinear flow functions are piecewise smooth. In practice, it is impossible to get an analytic solution, which requires the use of multiple linear regression algorithms. Therefore, linearization operations have to be conducted. For example, the part  $q_{12}(X_1(k), X_2(k))$  after linear regression can be written as  $a_{12}X_1(k) + b_{12}X_2(k)$ , where  $a_{12}$  and  $b_{12}$  as regression coefficients can be obtained by the unconstrained nonlinear multivariable optimization method (in this work `fminunc` function in Matlab was used to obtain the coefficients). The approximated linear functions used for

MLD are presented in the following:

$$X_1(k+1) = \frac{T_s}{A_{s1}} [q_{b1-d}(k) + q_{w1-d}(k) + q_{c1-d}(k) - a_{12}X_1(k) - b_{12}X_2(k)] + X_1(k) \quad (18)$$

$$X_2(k+1) = \frac{T_s}{A_{s2}} [-a_{yz_2}Z_2(k) - b_{yz_2}Y_{ZL_2}(k)\delta_2(k) + q_{w2-d}(k) + q_{c2-d}(k) + a_{12}X_1(k) + b_{12}X_2(k) - a_{23}X_2(k) - b_{23}X_3(k)] + X_2(k) \quad (19)$$

### 2.3 Optimization

For the optimization, the objective function and some constraints have to be defined. There are two kinds of objectives,  $J_i(\tilde{x}_i(k))$  and  $J_k(\tilde{\delta}_k(k))$ .  $J_i(\tilde{x}_i(k))$  is the objective function for water level of each virtual tank, the water level have to stay in the safety region,  $r_{i1}$  is the unsafe level,  $r_{i2}$  is the dangerous level, which can cause flood disaster.  $\lambda_{i1}$ ,  $\lambda_{i2}$ ,  $\lambda_{i3}$  are cost factors,  $e_{max,i1}(k)$  and  $e_{max,i2}(k)$  are the exceedances of reference water level  $r_{i1}$  and  $r_{i2}$  respectively;  $e_{i3}(k)$  is a binary auxiliary variable, which shows whether or not the water level has been exceeded.

$$J(k) = J_1(\tilde{x}_1(k)) + J_2(\tilde{x}_2(k)) + J_3(\tilde{x}_3(k)) + J_4(\tilde{x}_4(k)) + J_5(\tilde{x}_5(k)) \quad (20)$$

$$J_i(\tilde{x}_i(k)) = \lambda_{i1}e_{max,i1}(k) + \lambda_{i2}e_{max,i2}(k) + \lambda_{i3}e_{i3}(k) \quad (i=1,2,3,4,5) \quad (21)$$

$$e_{max,i1}(k) = \max(\max(\tilde{x}_i(k)) - r_{i1}, 0) \quad (22)$$

$$e_{max,i2}(k) = \max(\max(\tilde{x}_i(k)) - r_{i2}, 0) \quad (23)$$

$$e_{i3}(k) = \begin{cases} 1 & \text{if } e_{max,i2}(k) > r_{i2} \\ 0 & \text{if } e_{max,i2}(k) \leq r_{i2} \end{cases} \quad (24)$$

The minimization of this cost function is reformulated into the following equivalent form:

$$\min J_i(\tilde{x}_i(k)) = \min (\lambda_{i1}e_{max,i1}(k) + \lambda_{i2}e_{max,i2}(k) + \lambda_{i3}\delta_{i3}(k)) \quad (25)$$

$J_k(\tilde{\delta}_k(k))$  represents the objective function for sluices. The first term denotes the cost of closing the sluices, the second term stand for the cost of changing the state.

$$J_k(\tilde{\delta}_k(k)) = \lambda_{k1} \sum_{j=1}^N [1 - \delta_k(k+j-1)] + \lambda_{k2} \sum_{j=1}^N |\delta_k(k+j-1) - \delta_k(k+j-2)| \quad (26)$$

The minimization of the first part is rewritten as follows:

$$\min \lambda_{91} \sum_{j=1}^N [1 - \delta_9(k+j-1)] = \min -\lambda_{91} \sum_{j=1}^N \delta_9(k+j-1) \quad (27)$$

The minimization of the second part is :

$$\min \lambda_{92} \sum_{j=1}^N |\delta_9(k+j-1) - \delta_9(k+j-2)| = \min \lambda_{92} \sum_{j=1}^N p_j \quad (28)$$

$\lambda$  is a weighting factor,  $p_j$  are continuous auxiliary variables. Considering the objective functions and constraints, the optimization problem can be solved by the MILP (linear programming mixed-integer) method.

$$\min \mathbf{c}^T \boldsymbol{\theta} \quad (29)$$

$$G\theta \leq h \quad (30)$$

where  $C$  is a matrix of the weight coefficients;  $\theta$  is the matrix of the optimal target factors.

### 3. CASE STUDY

The area of the case study is located in the Yangtze River Delta in Middle East of china. The area has a flat terrain, with dense river network, and each river (Channel) has more than one water conservancy project. North of the area is the Yangtze River. The water level of Yangtze River is affected by the upstream runoff and tide factors of the downstream. Therefore, along the Yangtze River not only dams were built for the flood protection, but also in the crossing between local rivers and the Yangtze River a set of gates were constructed for the management of navigation and controlling of the river water level, and exchanging the water between local rivers and the Yangtze. In some special positions two-way water pump stations were constructed for flood and drought control. Within the local river network, different kinds of sluices were constructed for the easy management of wateruse. Because of the artificial structure, the natural water system has evolved into a complex hybrid water system. In this case study, the model region is divided into 19 virtual tanks, and contains six Yangtze River gates, eight river control sluices and a pumping station (consist of five 20 m<sup>3</sup>/s two-way pumps).

#### 3.1 Scenario description

The rainfall forecast for the next three days from 09:00 on 10.Aug.2016 to 15:00 on 13.Aug.2016 shows a lot of rain (especially, there is a heavy rainfall with 210 mm on 10.Aug.). Figure 4 gives the overview of the rainfall situation.

The control goals are shown in Table 1.

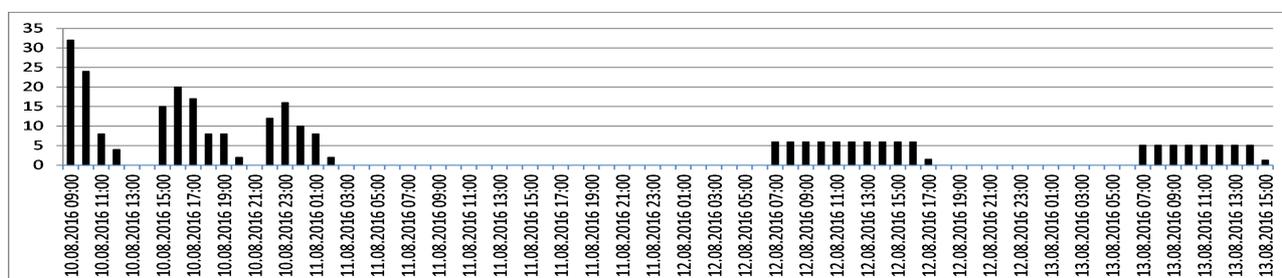


Figure 4. Precipitation forecast

Table 1. The control goals

Submodel	Risk water level	Flood water level
Virtual Tank 03	3,4 m	4,0 m
Virtual Tank 04	3,3 m	4,2 m
Virtual Tank 05	3,4 m	4,1 m

The whole system was compiled and run under the MATLAB environment. The simulation model shows you how the water level changed through the control from experience (see Figure 5, the dotted line). It shows very clearly that after one day of the heavy rainfall the water level can cause a flood, the operation plan is not suitable for this heavy rain event, it has to be optimized.

The solid line in Figure 5 shows the new operation plan for Gate(YZ-03) using the optimization model. There are two significances in this figure: on one hand, it is very clear that the water situation using the optimization control is be much better than from the expert experience, the risk of flood would be belayed about 8 hours (look the quadrangle), in this time humans can take a lot of

counter measures and actions against the flood; and on the other hand, the optimization calculate within a defined time period, in this time period the water level could be also reached the risk level of flood, therefore, the optimization process should be renewed by the actual water situation, so that the modified optimized control plan would be suggested.

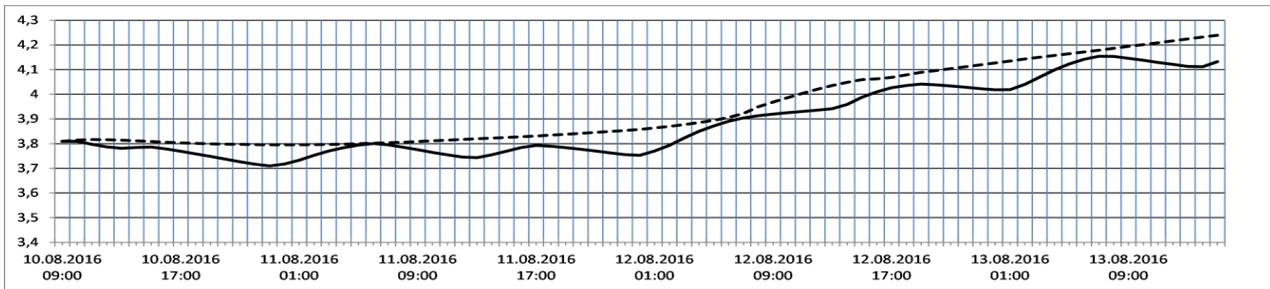


Figure 5. Water level on A03 with empirical operation and optimization

## 4. CONCLUSIONS

In this study, the model of a hybrid water system based on Visual Tank concept is established. The theory of the solution for bi-directional flow in hybrid water system is discussed. A water resources scheduling optimization system for hybrid water system was developed based on MPC-MLD optimization method. MPC-MLD is a well-known approach from cybernetics area, but very fresh application in hybrid water systems. This method can support the target of flood control, drought or environmental diversion in complex river networks for in short-term period. This work provides a new method to solve the optimization scheduling problem. In the future research, the special water policy of water resources protection will be implemented in MPC-MLD method. The optimization will be made more efficient und fit for the practical applications.

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