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Assessment of bedload transport in gravel-bed rivers with a new fuzzy adaptive regression

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Abstract: Bedload transport in gravel-bed rivers exhibits notable differences with increasing discharge. For low discharges, the bedload consists mainly of sand and fine gravel, originating from the upstream, while the surface armor layer remains intact. For high discharges, approximately for bankfull flows, the armor layer breaks up and the bedload, which increases considerably, comprises gravel from the surface layer as well as finer material from the subsurface layer. However, this transition occurs gradually, and for this range of discharges the contribution of each bedload transport mechanism is unclear. An adaptive fuzzy based regression is proposed in this article to include this no constant behaviour of the discharge-bedload relation. For low discharges, until a fuzzy threshold, a conventional (crisp) relation between bedload transport and discharge is established, while for high discharges, a curve with sharper slope must be derived to fit the bedload data. Between these curves and for a discharge range where the armor layer gets disrupted, the Sugeno theory of expert systems is used, that is, each fuzzy rule with a corresponding curve holds to some degree. Instead of the usual trial and error method, a two-phase training optimization based method to achieve this adaptive curve is proposed. The training method is a couple between the heuristic Particle Swarm Optimization method and the conventional regression method. In the present study, the proposed methodology is implemented on discharge-bedload data from several gravel-bed rivers from mountain basins of Idaho, U.S.A.

Key words: Armor layer, bedload transport, fuzzy rules, fuzzy sets and logic, Particle Swarm Optimization

1. INTRODUCTION

Sediment transport in steep gravel-bed streams is a complicated phenomenon that needs to be quantified for engineering design, habitat protection, and river management and restoration, among others. However, sediment transport prediction within tolerable error margins in such streams remains highly problematic (Gomez and Church, 1989) and site-specific (Barry et al., 2004; Kitsikoudis et al., 2014), and constitutes a challenge for engineers and geomorphologists. This is particularly owed to the fact that the incipience of sediment motion may occur for a relatively wide range of flow conditions and there is not a single threshold for specified flow conditions (Buffington and Montgomerry, 1997; Kitsikoudis et al., 2016). This is attributed to the near-bed turbulence (e.g. Diplas et al., 2008), sediment exposure and protrusion to the flow (e.g. Kirchner et al., 1990), and relative depth (e.g. Recking, 2009).

Natural gravel-bed rivers are usually poorly sorted and characterized by a wide range of sediment sizes. These rivers tend to self-organize and form an armor layer, which is a coarse surface layer overlying a finer sublayer (Parker and Klingeman, 1982). This has direct implications on the boundary layer hydrodynamics and, consequently, the bedload transport. In general, the break-up of the armor layer leads to different phases of bedload transport with distinct sediment properties. Ryan et al. (2002) implemented a piecewise regression to show these different bedload transport trends. For flows that cannot entrain the coarse armor layer, the bedload comprises sand and fine gravel from the interstices among the coarse grains or from sediment patches. For intense flows that occur in floods, the armor layer breaks up and the bedload consists of the coarse particles that constitute the armor layer and the finer sediment from the subsurface layer. Ryan et al. (2002)

marked the break-up of the armor layer with a single point, which causes an abrupt change in the sediment transport trend, to facilitate the calculations. However, the break-up of the armor layer is a phenomenon that occurs gradually. Firstly, the coarse particles that are more exposed to the flow and are subject to increased turbulent shear stresses, are more likely to be entrained. As the flow gets more intense, the probability of entrainment is increased for particles less exposed to the flow, until an upper limit where the whole surface layer is in motion. In this article, an adaptive fuzzy based regression is proposed to include this no constant behaviour. For low discharges, until a fuzzy threshold, a conventional (crisp) relation between bedload transport and discharge is established, while for high discharges, a curve with sharper slope must be derived to fit the bedload data. Between these curves and for a discharge range where the armor layer gets disrupted, each fuzzy curve holds to some degree. Therefore, it can be suggested that the proposed method emanates from the physical problem itself.

2. PROPOSED ARCHITECTURE OF THE FUZZY RULE BASED SYSTEM

A widely used form of the fuzzy rule based system is the following:

Ru (λ): If x_1 is A_1^λ and x_2 is A_2^λ … and x_N is A_N^λ

then $\quad y_\lambda = a_{\lambda 0} + a_{\lambda 1}x_1 + \ldots + a_{\lambda N}x_N \qquad (1)$

where A_i^λ is a fuzzy set in Universe $U_i \subset R$, λ is the order of the rule, $(x_1, x_2, \ldots, x_N)^T \in U$ are the input variables. Let L be the number of the fuzzy rules, that is, $\lambda = 1, \ldots, L$ and let N be the number of the independent variables. By using the extension principle and the weighted average defuzzification formula, the final single output is (Chen and Pham, 2001; Botzoris and Papadopoulos, 2015):

$$y = \frac{\sum_{\lambda=1}^{L}\left[\left(\mu_{1,\lambda}(x_1) \wedge \ldots \wedge \mu_{N,\lambda}(x_N)\right)\left(a_{\lambda 0} + a_{\lambda 1}x_1 + \ldots + a_{\lambda N}x_N\right)\right]}{\sum_{\lambda=1}^{L}\left(\mu_{1,\lambda}(x_1) \wedge \ldots \wedge \mu_{N,\lambda}(x_N)\right)} \qquad (2)$$

where μ is the membership function which characterizes the fuzzy set and takes values into the closed interval between zero and one (Tsakiris et al., 2006). As $\wedge$, the fuzzy intersection is symbolized.

Many scientific articles are based on the Matlab toolbox which is based on the ANFIS systems. However, even if the errors remain within acceptable range, sometimes the rational and logical basis of the application is erroneous (Sen, 2010). In contrast, in the proposed method, the problem itself modulates the structure of the expert system. Indeed, by using the fuzzy logic, the couple between the linguistic variables and numerical methods can be achieved and hence, more smart numerical models can be established.

In the proposed model, only one variable has the ability to derive the use of the proper linear relation. Thus, this variable (discharge) takes only two linguistic values (high and low) which correspond to quantitative fuzzy sets. In addition, in this application, only two rules exist (one with each linguistic value). Consequently, there are two regions without uncertainty where only one regression equation is activated. However, between two crisp regions (Fig. 1) there is a grey region where both rules are activated to some degree. Furthermore, a semi-static structure is supposed. That is, even if the starting points and the ending points are unknown, the shape of the membership functions μ_1 and μ_2 will follow the structure of Fig. 1. Based on Fig. 1, it is obvious, that only two rules are modulated, whilst in fact the grey region (fuzzy region) is between β_1 and β_2. In the middle of the distance between the parameters, the two membership functions μ_1 and μ_2 are equal to 0.5.

Let only one input variable (discharge), x. For given variables β_1 and β_2, the final output y for the input x is given as follows:

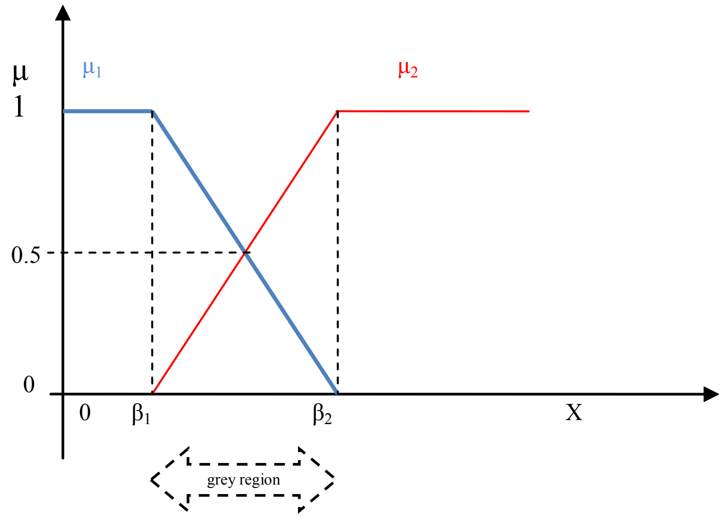

Figure 1. Architecture of the fuzzy rules

$$y = \frac{\mu_1(x)(a_{10}+a_{11}x)+\mu_2(x)(a_{20}+a_{21}x)}{\mu_1(x)+\mu_2(x)} = \frac{\mu_1(x)}{\mu_1(x)+\mu_2(x)}a_{10} + \frac{\mu_2(x)}{\mu_1(x)+\mu_2(x)}a_{20} + \frac{\mu_1(x)x}{\mu_1(x)+\mu_2(x)}a_{11} + \frac{\mu_2(x)x}{\mu_1(x)+\mu_2(x)}a_{21} \quad (3)$$

The produced smart regression model is suitable to study the bedload transport in gravel-bed rivers which exhibits notable differences with increasing discharge. For low discharges, the bedload discharge consists mainly of sand and fine gravel, originating from the upstream, while the surface armor layer remains intact. For high discharges, approximately for bankfull flows (Ryan et al., 2002), the armor layer breaks up and the bedload, which increases considerably, comprises gravel from the surface layer as well as finer material from the subsurface layer. However, this transition occurs gradually, and for this range of discharges the contribution of each bedload transport mechanism in the total bedload transport is unclear.

The training of the proposed model is achieved based on a two-phase approach. Let the thresholds β_1 and β_2 be known. Assuming that a set of M input-output data is given, the following linear system of algebraic equations is concluded:

$$\mathbf{\Lambda}\cdot\mathbf{\theta} = \mathbf{b}$$

$$\mathbf{\Lambda} = \begin{bmatrix} \frac{\mu_1(x^1)}{\mu_1(x^1)+\mu_2(x^1)} & \frac{\mu_2(x^1)}{\mu_1(x^1)+\mu_2(x^1)} & \frac{\mu_1(x^1)x^1}{\mu_1(x^1)+\mu_2(x^1)} & \frac{\mu_2(x^1)x^1}{\mu_1(x^1)+\mu_2(x^1)} \\ & & \cdots\cdots & \\ \frac{\mu_1(x^M)}{\mu_1(x^M)+\mu_2(x^M)} & \frac{\mu_2(x^M)}{\mu_1(x^M)+\mu_2(x^M)} & \frac{\mu_1(x^M)x^M}{\mu_1(x^M)+\mu_2(x^M)} & \frac{\mu_2(x^M)x^M}{\mu_1(x^M)+\mu_2(x^M)} \end{bmatrix}$$

$$\mathbf{\theta} = [a_{10} \quad a_{20} \quad a_{11} \quad a_{21}]^T$$

$$\mathbf{b} = [y_1 \quad \ldots \quad y_M]^T \quad (4)$$

Based on the usual least squares method, the optimal coefficients **θ*** can be determined:

$$\boldsymbol{\theta}^* = \left[\boldsymbol{\Lambda}^{\mathrm{T}} \cdot \boldsymbol{\Lambda}\right] \boldsymbol{\Lambda}^{\mathrm{T}} \mathbf{b} \tag{5}$$

The successfulness of the model is measured on the basis of the standard error between the observed and the predicted data resulting from the applied model:

$$E = \left(\mathbf{b} - \boldsymbol{\Lambda} \cdot \boldsymbol{\theta}^*\right)^{\mathrm{T}} \cdot \left(\mathbf{b} - \boldsymbol{\Lambda} \cdot \boldsymbol{\theta}^*\right) \tag{6}$$

Thus, in case that the thresholds β_1 and β_2 are known, the constant terms of the linear relations can be determined by following a least squares approach.

The entire training method is a couple between the heuristic Particle Swarm Optimization (PSO) method and the conventional regression method. The application of the conventional regression method is presented above. According to the first phase, a population of possible solutions is modulated. Each possible solution contains only the unknown thresholds β_1 and β_2. For each candidate solution (the values of β_1 and β_2), the above analysis of the usual regression model is running and hence, the optimal coefficients and the standard error E are calculated. Finally, the thresholds with the minimum error (Eq. 6) are chosen after a selected number of iterations.

3. PARTICLE SWARM OPTIMIZATION METHOD AND THE PROPOSED LEARNING PROCESS

Particle Swarm Optimization (PSO) is a stochastic global optimization method based on the simulation of social behavior. As in Genetic Algorithms (GA), PSO exploits a population of candidate solutions to probe the search area. Therefore, the Particle Swarm Optimization (PSO) algorithm can be characterized as one of the population based algorithms (Parsopoulos and Vrahatis, 2002). Although PSO is an effective and widely used method, it is also much simpler than several other evolutionary algorithms (e.g. GA), due to the fact that it does not contain the crossover and mutation operations in the original version (Spiliotis, 2014).

PSO can deal with nonlinear optimization problems in non-convex domains (Ostadrahimi, 2012). Each candidate solution is called a particle, and the set of potential solutions in each iteration creates the '*swarm*' (Papadopoulos et al., 2011; Spiliotis et al., 2016). A '*swarm*' has a dimension N', in which N' is the number of the potential solutions. Each potential solution is comprised of D variables, in which D is the dimension of the problem (Spiliotis et al., 2016).

More analytically, the population dynamics in PSO simulates the behavior of a "*birds flock*", where social sharing of information takes place and individuals benefit from the discoveries and previous experience of all other companions during their search for food. Thus, two variants of the PSO algorithm were developed considering either a local neighbourhood or a global neighbourhood. In the former, the partial optimum of the particle is usually applied (Spiliotis, 2014). In the latter, each particle moves towards its best previous position and towards the best particle in the whole swarm (Eberhart et al., 1996; Parsopoulos and Vrahatis, 2002; Spiliotis et al., 2016).

Several modifications to the original version of the PSO method of Kennedy and Eberhart (1995) have been proposed, such as the adaptation of inertia term and the consideration of the maximum velocity (e.g. Poli et al., 2007). The basic PSO algorithm is presented below (e.g. Shi et al., 1998; Poli et al., 2007):

Algorithm Original PSO

1: Initialize a population array of particles with random positions and velocities on D dimensions in the search area.

2: *Loop*

3: For each particle, evaluate the desired optimization fitness function in D variables.

4: Compare particle fitness evaluation with its best previously visited position (p_i). If the current value is better than p_i, then set p_i equal to the current value.

5: Identify the particle in the neighbourhood with the best success so far, and assign its index to the variable p_g.

6: Change the velocity and position of the particle according to the following equation

$$\begin{cases} \vec{\upsilon}_i\left(t+1\right) = \omega\vec{\upsilon}_i + c_1\rho_1\left(\ \right)\cdot\left(\vec{p}_i - \vec{x}_i\left(t\right)\right) + c_2\rho_2\left(\ \right)\cdot\left(\vec{p}_g - \vec{x}_i\left(t\right)\right) \\ \vec{x}_i\left(t+1\right) = \vec{x}_i\left(t\right) + \vec{\upsilon}_i\left(t+1\right) \end{cases} \tag{7}$$

7: If a criterion is met (usually a sufficiently good fitness or a maximum number of iterations), exit loop.

8: *End loop*

where $\rho()$ is a vector of random numbers uniformly distributed in the open interval 0,1 that is generated at each iteration and for each particle, p_i is the best previously visited position of the i^{th} particle (partial optimum) and p_g is the global best previously visited position of all particles (global optimum). Furthermore, the term $c_1\rho_1\left(\ \right)\cdot\left(\vec{p}_i - \vec{x}_i\right)$ that associates the particle's own experience with its current position, is weighted by the constant c_1, and is called individuality (cognitive acceleration). The term $c_2\rho_2\left(\ \right)\cdot\left(\vec{p}_g - \vec{x}_i\right)$ is associated with the social interaction between the particles of the swarm and weighted by the constant c_2, and is called sociality (social acceleration). For the standard PSO, the coefficients c_1 and c_2 were allowed to take values in the interval 1.5 to 2.5 (Papadopoulos et al., 2011; Spiliotis et al., 2016).

To summarize the learning process, at first, a swarm of possible solutions is randomly created. Each possible optimum solution (member of the swarm) contains only the values of the β_1, β_2 parameters of the membership functions. Then, based on the architecture of the membership functions of Fig. 1, the training process is activated, and the values of the constant terms of the regression curves, $\boldsymbol{\theta}^*$ and the error function, E, are finally determined. Subsequently, the algorithm returns to the PSO process in order to compare the solutions and to re-find the global optimum and the partial optimum for each possible solution. Finally, the new position of each member of the swarm is determined and the process is repeated for the new swarm.

The process is finished when the maximum number of iterations occurs. The reason for which a conventional optimization toolbox is not used, is that the formulation of the two rules follows a very no conventional form. The membership functions are not differentiable functions at the critical points (β_1 and β_2).

4. CASE STUDY

The proposed methodology is tested in several mountainous gravel-bed streams from Idaho, U.S.A., with the utilized data originating from an extensive field campaign (King et al., 2004). Firstly, the bedload data for Big Wood River are studied. A curve between the bedload transport rate, mg (output variable) and the discharge, Q (input variable) is achieved based on the proposed methodology with two fuzzy rules and the architecture of Fig. 1. Practically, from Fig. 2 it is evident that the optimum heuristically based solution is achieved after the 10^{th} iteration. According to the proposed methodology, the square error, E, is equal to 8.9721 which is significantly smaller than the square error of the usual linear regression (E= 17.2510).

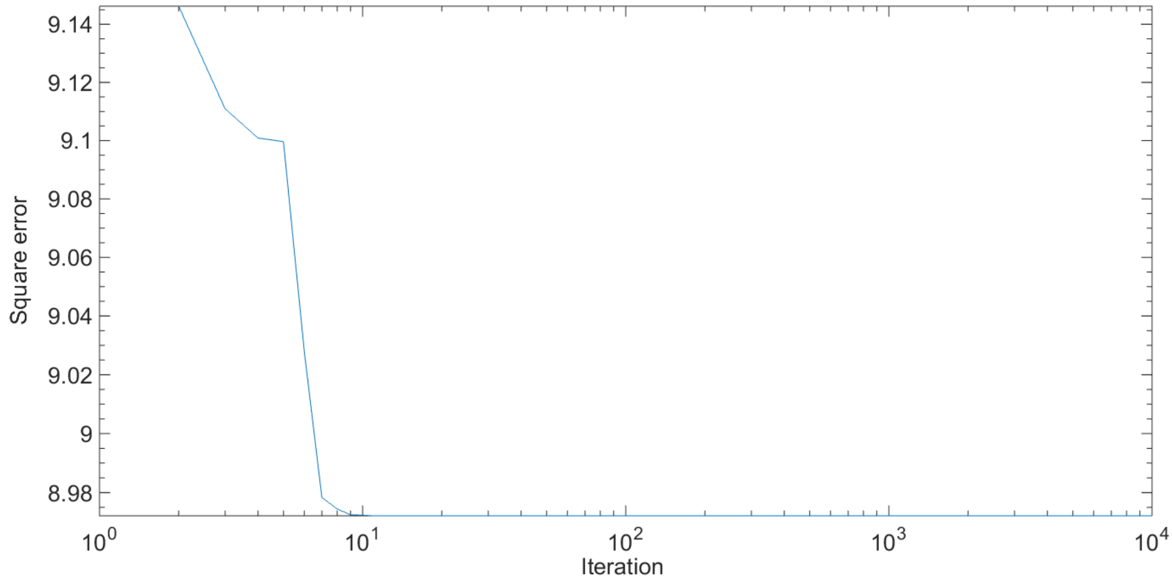

Figure 2. Squared error as a function of the number of iterations for Big Wood River

The coefficient of efficiency E' proposed by Nash and Sutcliffe (1970) is defined as one minus the sum of the squared differences between the predicted and observed data normalized by the variance of the observed values (Krause et al., 2005):

$$E' = 1 - \frac{\sum_{i=1}^{M} \left(O_i - mg_i\right)^2}{\sum_{i=1}^{M} \left(O_i - \bar{O}_i\right)^2} \tag{8}$$

where M is the number of data, the symbol O refers to the observed data (bedload transport rate), while the symbol mg to the predicted data.

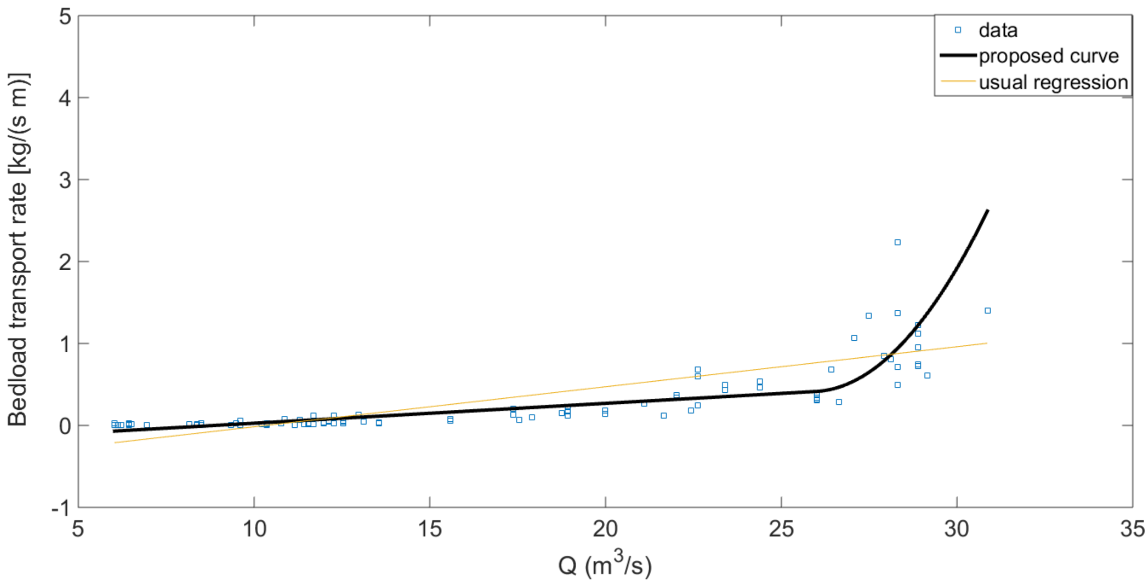

Figure 3. Data, usual crisp linear regression and the produced (crisp) curve for Big Wood River

The error E' is equal to 0.7183 which is close to unity (perfect result). The thresholds β_1 and β_2 are equal to 25.9952 m^3/s and 30.8654 m^3/s. The (crisp) relation between the bedload transport rate, mg, and the discharge, Q, is the following:

$$mg(Q) = \frac{\mu_1(Q)}{\mu_1(Q)+\mu_2(Q)}(-0.2151) + \frac{\mu_2(Q)}{\mu_1(Q)+\mu_2(Q)}(-11.3331) + \frac{\mu_1(Q)}{\mu_1(Q)+\mu_2(Q)}0.0242 \cdot Q + \frac{\mu_2(Q)}{\mu_1(Q)+\mu_2(Q)}0.4519 \cdot Q \quad (9)$$

The first two terms are referred to the constant terms and the last two are referred to the coefficients of the independent variable. The two rules are fired according to the corresponding ratio of the membership functions. For example, for the first rule, it is fired according to the ratio: $\frac{\mu_1(Q)}{\mu_1(Q)+\mu_2(Q)}$.

In case of the Fourth of July Creek (Fig. 4), the parameters β_1 and β_2 are between 3.0076 m^3/s and 3.0599 m^3/s which means that the fuzzy area (where the behavior is changed) is rather small, and the efficiency coefficient of Nash and Sutcliffe is equal to 0.7755. However, the fact that the generated curve is not monotonic in case of the Fourth of July Creek, highlights the data-driven nature of this technique and the necessity for high quality representative data. The curves in Fig. 3 exhibit the expected monotonic form, contrary to Fig. 4. This is attributed to the fact that in the grey region, within the two linear regression curves, there are more available points for the proper training of the suggested model. This grey area corresponds to infrequent high discharges that induce the break-up of the armor layer.

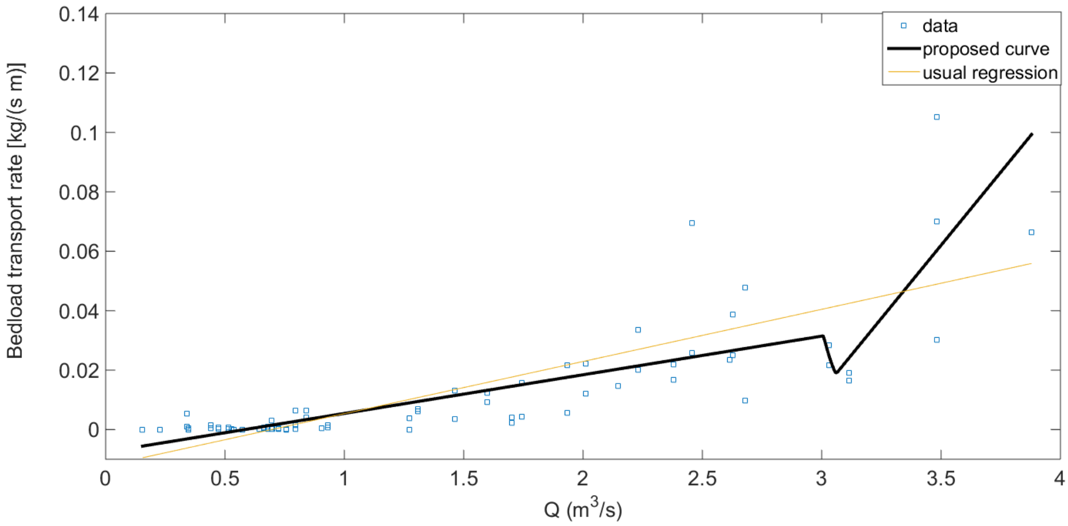

Figure 4. Data, usual crisp linear regression and the produced (crisp) curve for Fourth of July Creek

Another interesting point which must be clarified, is that the calculated curve is a crisp relation (without uncertainty as the fuzzy regression model of Tanaka (1987), see also Tsakiris et al., 2006; Kitsikoudis et al., 2016), although elements of fuzzy sets and logic are used to construct the model. In fact, the fuzziness is implemented in the grey region between the critical points.

5. CONCLUSION

A smart fuzzy regression is produced based on fuzzy rules (with a given architecture) combined with the conventional linear regression. The training is based on the coupling between the PSO and the conventional linear regression. The method is successfully applied in order to determine the bedload transport rate as a function of the discharge in selected gravel-bed streams. After several running processes it is concluded that many times the break-up of the armor layer can be identified between two parameters in case they are neighboured and hence the model can automatically detect such a physical process. However, this does not happen in all cases, even if the training process can be characterized as successful.

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