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Application of Bayesian algorithm in continuous streamflow modeling of a mountain watershed

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Abstract: Appropriate prediction of streamflow at mountain region poses substantial challenges to any hydrology modelling system. In this type of watershed, the rainfall-runoff process is typically rapid; thus, a key approach for a successful simulation is how to reduce the uncertainty and error in simulation. The focus of this study is on parameter and predictive uncertainty estimation of a semi-distributed hydrology model (i.e., ARNO). The ARNO model is coupled with the DiffeRential Evolution Adaptive Metropolis (DREAMzs) algorithm to calibrate fifteen hydrological parameters. Analysis suggests that DREAM efficiently reduced the uncertainty associated with modelling and performed best during moderate to moderately high flow. The 95 prediction uncertainty (95 PPU) bracketed most high and medium flows. Overall, application of DREAM algorithm to the ARNO model identified the uncertainty associated with the maximum soil moisture storage and the routing model components. ARNO parameter uncertainty largely depends on the complexity, and variability of streamflow records, and the simplicity of its lumped formulation. These results provide useful insights into the complexities of surface flow simulation and demonstrate the importance of reducing parameter and predictive uncertainty of a rapid response hydrologic system.

Key words: Parameter Uncertainty Assessment, ARNO, DREAMzs, Continuous Simulation, Mountainous Watershed

1. INTRODUCTION

Rainfall-runoff models are widely used in hydrology to simulate river basin water quantity and play an important role in management of water resources. It is an accepted fact that a hydrological model prediction, should not be deterministic, most-probable representation, but should also explicitly include an estimate of uncertainty. Uncertainty in model predictions arise from measurement errors associated with the system input (forcing) and output, model structure errors due to the aggregation of spatially distributed real-world processes into a mathematical model, initial conditions and measurement errors cause by in situ observations that can be collectively considered as data errors. Realistic assessment of these various sources of uncertainty provide reliable information to decision makers and practitioners and help to manage water resources in different hydrological systems.

In order to estimate predictive uncertainty of the hydrologic models, infer the parameters, and predict model outputs, various methodologies may be adopted, including first-order approximation, state-space filtering, multi model averaging, and various Bayesian approaches. Among these approaches, Bayesian algorithms have been widely used in hydrology for statistical inference of parameters and model predictive uncertainty. Under Bayes theorem, posterior distribution combines the data likelihood with the prior distributions of parameters, narrow parameter space and predict the uncertainty.

Recently, a new Markov chain Monte Carlo (MCMC) sampler, namely DREAMzs (DiffeRential Evolution Adaptive Metropolis algorithm), was used under a Bayesian framework as an efficient and robust sampler in hydrologic modeling. Compared to the generalized likelihood uncertainty estimation (GLUE), the main advantage of DREAM (using MCMC simulation) is separating the effects of input (forcing), parameters and model structural uncertainties from total predictive uncertainty (Vrugt et al., 2009b). DREAMzs is based on the original DREAM algorithm (Vrugt et al., 2009a) that was modified for an efficient estimation of the posterior probability density function of parameters o to address high-dimensional posterior exploration problems.

This study linked DREAMzs with for a semi-distributed conceptual continuous rainfall-runoff model (ARNO). The ARNO model, which derives its name from its first application to the ARNO River, is a semi-distributed conceptual continuous rainfall-runoff model. The soil moisture balance module of this model is taken originally from the Xinanjiang model (Zhao, 1977), in which the spatial distribution of the soil moisture capacity is expressed in the form of a probability distribution function. Later, the original Xinanjiang model scheme was modified by Todini (1988), by allowing the soil moisture to be depleted not only by evapotranspiration, as in the original Xinanjiang model, but also by drainage into the river network and percolation into the water table. This modified ARNO model were adopted by several scholars (e.g. Franchini, 1996; Khazaei et al., 2011 & 2013; Adams, 2015).

This study makes a comprehensive evaluation about the parameter and predictive uncertainty of daily streamflow in mountainous catchment from Iran. Special attention is paid: 1) model setup, parameterization and choice of calibration parameters, 2) definition of performance criteria and likelihood function, and 3) prediction of parameter and predictive uncertainty for daily streamflow simulation of the ARNO model. We are especially concerned with identifying potential regionalization relationships, correlating parameter sensitivity and uncertainty with site characteristics and analyzing posterior probability density functions of sensitive parameters.

2. METHODOLOGY

2.1 Study area

The Karaj River Basin (KRB) is located in the mountainous region of Alborz province (51°03′ to 51°36′ longitude and 35°53′ to 36°10′ latitude) with a drainage area of 840 sqkm. AmirKabir Dam (Karaj Dam), one of the most important water resources of Tehran province, has been constructed on the Karaj River. Karaj River annual mean flow rate at its entrance to the Karaj Dam is 450×106 m3; additionally 8.2 and 1450 CMS (cubic meter per second) in this location were measured, respectively as minimum and peak annual temporal flow rates. This dam supplies water for municipal and cultural uses and electrical energy generation. It has been located 23 km north of Karaj and 63 km northwest of Tehran, and is under operation since 1963.

2.2 Hydrology model

ARNO continuous rainfall-runoff model developed by Todini, (1988) and is used to simulate daily runoff in this study. This model is broadly used in water management, low flow, and real-time flood forecasting studies in different parts of the world. In ARNO, the basin is composed of an infinite number of elementary areas, each of which has a different soil moisture capacity. The spatial distribution of the soil moisture capacity is expressed in the form of a probability distribution function. Soil moisture content is fed by rainfall that infiltrates into the soil and is depleted by evapotranspiration, drainage and percolation. For each of the elementary areas with different soil moisture capacity and different soil moisture content, the continuity of mass is simulated over time. Basin runoff is the integral of the runoff of elementary areas, transferred to the outlet of the basin via a routing module (Todini, 1996). The upper and the lower bounds that define the prior uncertainty ranges of the ARNO parameters are listed in Table 1. More details on the ARNO model are described by Todini (1996).

Table 1. The ARNO parameters used in this study

Parameter's Number	Parameter	Dimension and Symbol	Lower Bound	Upper Bound
1	Convectivity coefficient for the Hillslope	C1 [m/s]	1	3
2	Diffusivity coefficient for the Hillslope	D1 [m^2/s]	10	1000
3	Length of the path traveled by water to reach the channel	DX1 [km]	10	100
4	Convectivity coefficient for the channel	C2 [m/s]	1	3
5	Diffusivity coefficient for the channel	D2 [m^2/s]	100	10000
6	Length of the channel reach	DX2 [km]	1000	100000
7	Average volume of soil moisture storage	Wm [mm]	50	600
8	Moisture content threshold value in drainage calculation	SOL (or Wd) [mm]	0	300
9	Moisture content threshold value below which the percolation is negligible	SOL1 (or Wi) [mm]	0	100
10	A shape factor for the curve of soil moisture vs saturated areas	b	0.01	1
11	Maximum drainage that should be expected when the soil is completely saturated	Dmax [mm per time unit]	0	10
12	A drainage parameter	Dmin	0	10
13	Maximum percolation should be expected when the soil is completely saturated	PERC (or Is) [mm per time unit]	0	5
14	Exponent used to represent drainage when saturation is not reached	CESP (or c)	1.5	5
15	Initial volume of soil moisture storage	W0 [mm]	0	10

2.3 DREAMzs algorithm

DREAMzs, proposed by Schoups and Vrugt (2010), is based on the original DREAM algorithm (Vrugt et al. 2009a). This newest approach uses sampling from an archive of past states to generate candidate points in each individual chain and requires only three parallel chains to summarize the posterior distribution. DREAMzs does not require outlier detection and removal; it maintains a detailed balance at every single step in each of the parallel chains (Schoups and Vrugt 2010). In addition, DREAMzs contains a snooker update to generate jumps beyond parallel direction updates (ter Braak and Vrugt 2008) in order to increase the diversity of the candidate points. The snooker axis runs through the states of two different chains, and the orientation of this jump is different from the parallel direction update, utilized in DREAM (Laloy and Vrugt 2012). The algorithmic implementation of the snooker update within the context of the differential evolution Markov chain (DE-MC) method was proposed by ter Braak (2006). More discussion on this algorithm is provided by Schoups and Vrugt (2010) and Laloy and Vrugt (2012).

2.4 Likelihood function of DREAMzs model

Performance of each set of parameters in predicting the observed model may be evaluated by a likelihood measure (He et al., 2010). In literature, many authors defined and evaluated various likelihood functions based on the maximum absolute residual (Keesman and van Straten 1989); inverse error variance with a shape factor (Beven and Binley 1992), an auto-correlated Gaussian error model (Romanowicz et al. 1994); Nash-Sutcliffe efficiency criterion with shape factor T, as well as the exponential transformation of the error variance with shaping factor T (Freer et al. 1996); the maximum likelihood function (Makowski et al. 2002); minimum mean square error (Wang et al. 2005) and normalized root mean square error (RMSE; Izady et al. 2015).

In order to assess the uncertainty of ARNO, Sum of Square Error (SSE) was chosen as the likelihood function (Eq. 1).

$$L(\theta_i \mid O) = \sum_{j=1}^{n} (O_j - Y_j(\theta_i))^2 \tag{1}$$

where θ_i is the ith set of parameters, $Y_j(\theta_i)$ is the jth type of model output (simulated stream flow) under θ_i set of parameters, O is the observed stream flow, Oj is the jth observation of O, N is the number of parameter sets, and M is the number of observations.

2.5 Performance Criteria

In this research, two indices were used to quantify the goodness of calibration/uncertainty performance: a P-factor (the maximum value is 100%), which is the percentage of data bracketed by a 95% prediction uncertainty band (95PPU; Vrugt et al., 2009b; Hamraz et al., 2015; Pourreza-Bilondi et al., 2016; Pourreza-Bilondi and Samadi, 2016; Nourali et al., 2016), and R-factor [or d-factor; Eqs. (2) and (3)], which is the average width of the uncertainty band divided by the standard deviation of the corresponding measured variable (the minimum value is zero; Abbaspour et al., 2007). Theoretically, the value for the P-factor ranges between 0 and 100%, while the R-factor ranges between 0 and infinity. A P-factor of 1 and R-factor of 0 is a simulation that exactly corresponded to measured data. The average thickness of the 95PPU band or the R-factor are estimated in every run and the best simulation can be judged based on the simulation with almost observed and modeled data located inside the 95% band with the highest P-factor and the least R-factor.

$$\bar{d}_x = \frac{1}{k}\sum_{i=1}^{k}(X_U - X_L) \tag{2}$$

$$R - Factor = \frac{\bar{d}_x}{\sigma_x} \tag{3}$$

where $\bar{d}_x$ denotes the average distance between the upper and the lower 95PPU; X_U and X_L represent the upper and lower boundaries of 95PPU; and σ_x is the standard deviation of the measured data. As all uncertainties in the conceptual model and inputs are reflected in the measurement, bracketing most of the measured data in the predictive 95PPU ensures that uncertainty is depicted by the parameter uncertainty (Pourreza-Bilondi et al., 2016).

The total uncertainty index (TUI) is also calculated based on the P-factor and R-factor for each flood event (Eq. (4)).

$$TUI = \frac{P_{Factor}}{R_{Factor}} \tag{4}$$

After running rainfall-runoff model with DREAMzs algorithm, simulated flow obtained from the best set of parameters (with maximum likelihood function or minimum objective function) compared with observed streamflow. Statistical indicators including RMSE (Eq. (5), Kling-Gupta Efficiency (KGE, Eq. (6)) (Gupta et al., 2009), and Nash- Sutcliffe (NSE, Eq. (7)) criteria were used to compare the performance of calibration.

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(Qo_i - Qs_i\right)^2} \tag{5}$$

$$KGE = 1 - \sqrt{(\mathrm{cc}-1)^2 + (\alpha - 1)^2 + (\beta - 1)^2} \tag{6}$$

$$NSE = 1 - \frac{\sum_{i=1}^{n} \left(Qs_i - Qo_i\right)^2}{\sum_{i=1}^{n} \left(Qo_i - \overline{Qo}\right)^2} \tag{7}$$

where Qo_i is the observed discharge, $\overline{Qo}$ is the mean value of observations, Qs_i is the simulated discharge, n is the number of observations, cc is the linear correlation coefficient between Qo and Qs, α is the ratio of standard deviation of Qs to standard deviation of Qo, and β is the ratio of the mean of Qs to mean of Qo. The smallest possible RMSE along with KGE and NSE close to 1, represent a better performance of likelihood function.

2.6 Convergence of the Sampled Chains

The convergence of a DREAMzs run can be monitored with the R statistic (Gelman and Rubin 1992), which determines convergence to the stationary posterior distribution. Based on the R statistic, DREAMzs was efficient at traversing the parameter space by quickly achieving convergence at a stationary posterior distribution (R-stat < 1.2). In other words, DREAMzs convergence for each of the ARNO parameters (including routing and basin parameters) was achieved after approximately 43,000 iterations.

2.7 Parameter Uncertainty and Posterior Distribution

Posterior density responses of the model parameters were visually investigated and the marginal posterior densities were constructed using fifteen model parameters (figure 1). This plot is created using the top 10 % of the total samples that retains as behavioral parameter sets. The x-axis in each graph is fixed to the prior range of each individual parameter to facilitate comparison of model optimized parameters.

Best parameters values is also recognized with blue symbol (×). Analysis suggests that allocation of some parameters such as C1 and C2; DX1 and DX2 possibly either arriving to similar distributions or at least very close to each other for several parameters. Also marginal distribution of several parameters (e.g. SOL, b and Dmin) showed normal distribution over time. Moreover, it can be seen that PERC parameter value is approximately close to zero meaning that much of overland flow contributed to surface runoff. Therefore, groundwater module had no contribution to runoff generation. High degree of variability in some model parameters, e.g. moisture content threshold value below which the percolation is negligible (Wi) and initial volume of soil moisture storage (W0), along with other channel and soil properties reflect the fact that these parameters were insensitive and contributed less uncertainty in modeling.

2.8 Predictive uncertainty

Figure 2 shows the observed data (dots), a 95 % parameter uncertainty due to posterior distribution of the parameter estimates (green shaded area) and a 95% hydrograph predictive uncertainty associated with the total uncertainty (greyshaded area) in the DREAMzs sampler. X-axis represents time equal to seven years from Sep-2002 to Oct-2008. Based on figure 2, 95% total prediction uncertainty ranges may bracket most of the observed flows.

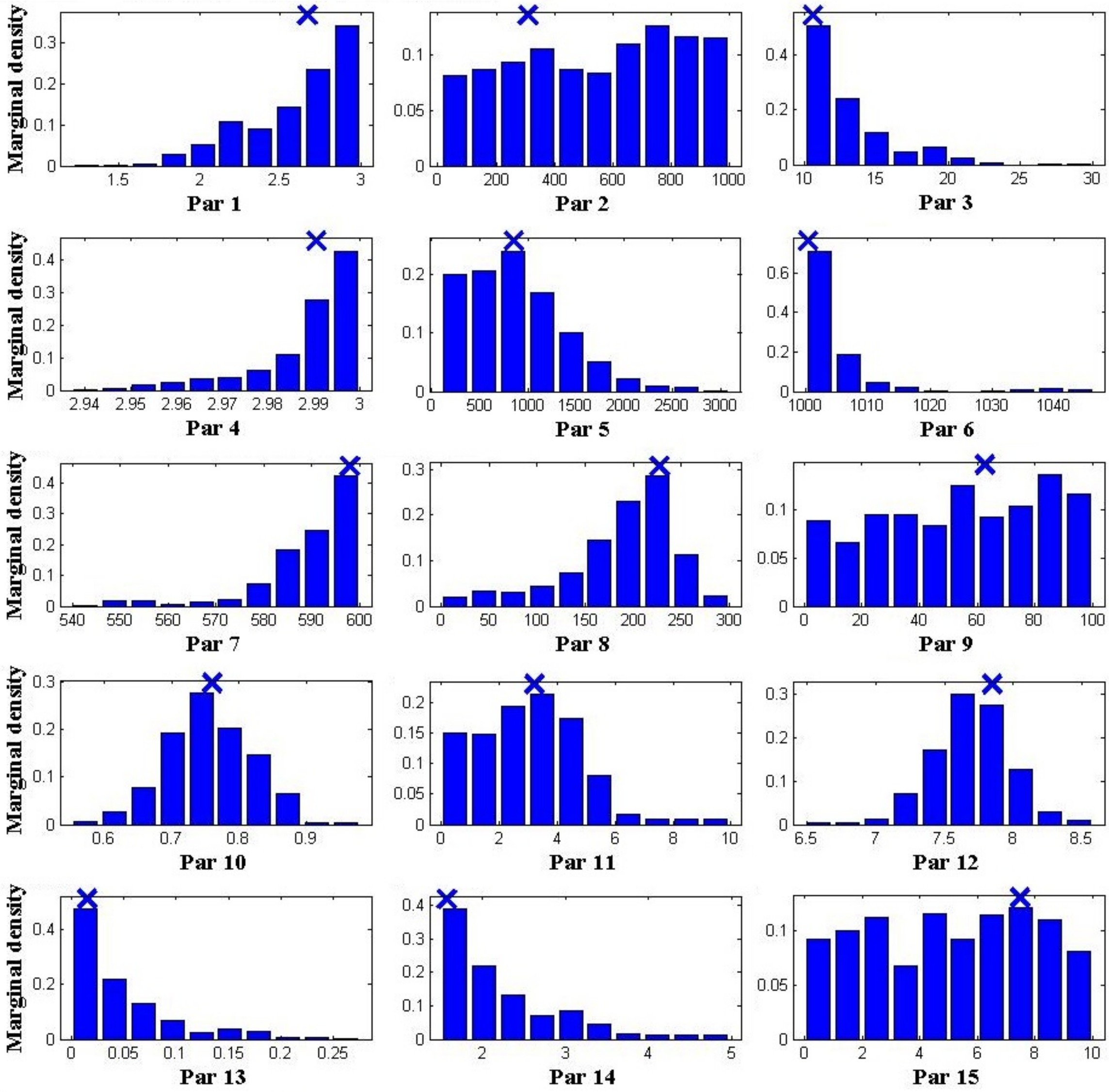

Figure 1. Marginal posterior probability distributions of fifteen ARNO parameters computed by DREAMzs

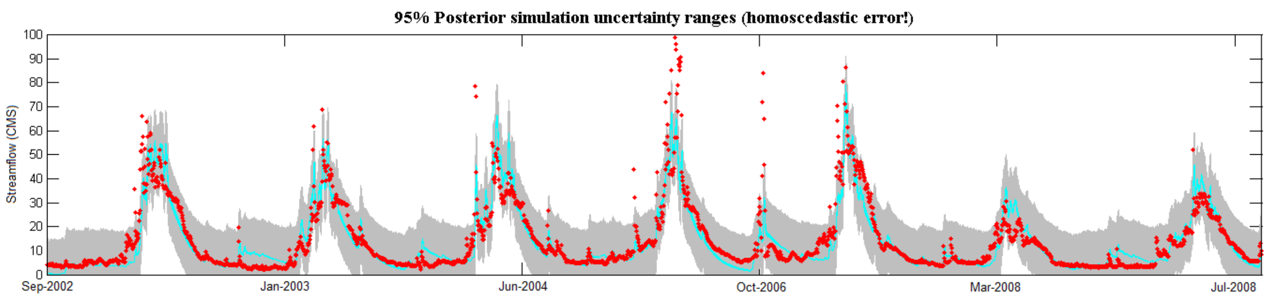

Figure 2. 95% total and parameter prediction uncertainty ranges of daily streamflow in the DREAMzs algorithm

Summary results and the performances of DREAMzs simulation are provided in Table 2. By calculating uncertainty assessment indicator (P-factor), 95% parameter uncertainty covers greater than 95% of observed data, but 95% total prediction uncertainty bounds are quite wide (figure 2) which is an indication of considerable uncertainty in the model structure and measured input data.

Table 2. Uncertainty and calibration performance criteria of the DREAMzs simulation

Uncertainty Criteria	Total Uncertainty	Parameter Uncertainty	Performance Criteria	Best Value
P-Factor%	96.83	7.08	KGE	0.81
R-Factor	1.92	0.05	RMSE	7.23
TUI	50.41	135.2	NSE	0.76

Green region in Figure 2 (corresponding to parameter uncertainty) does not bracket observed streamflow data (P-factor=7%). The gaps between the uncertainty bounds of parameter and the observed data may be originated from uncertainties in observational input–output data (forcing data) and structural inadequacy of model which is indicating that model structure and observation measurements are needed to be improved in order to achieve more accurate predictions as recommended by Laloy et al. (2010).

The R-factor and TUI values showed that the parameter uncertainty bound is highly efficient but it's not biased and skewed (P-Factor approximately 7%) that this could be due to disregarding rainfall data error while it may be dominant in many catchments (especially in mountainous regions) due to significant spatial and temporal variability of rainfall fields. It seems that rainfall forcing error should be considered in uncertainty assessment of ARNO model in future works. Ignoring errors of the input variables affects the structure of parameter uncertainty and confidence limits of the parameters in calibration of hydrologic models, and may lead to biased predictions (Kavetski et al., 2006). Overall, considering all performance criteria, including RMSE, KGE and NSE, results showed that DREAMzs algorithm represents a good performance in calibration and uncertainty assessment of the ARNO model in mountainous region. More research is needed to understand the rainfall-runoff processes in rapid response watershed. However, the outputs of this research will help to understand modeling deficiencies and improve simulation in this type of region.

Since ARNO was unable to skillfully capture the magnitude of base flow contribution, the authors argue that subsurface module of this model can be improved significantly to better simulate streamflow records in time of no rainfall events.

REFERENCES

Abbaspour, K. C., et al., 2007. Modelling hydrology and water quality in the pre-alpine/alpine Thur watershed using SWAT. J. Hydrol., 333(2): 413–430.

Adams, R., 1995. Assessing the performance of the NELUP hydrological models for river basin planning. Journal of Environmental Planning and Management, 38(1): 53-76.

Beven K, Binley A, 1992. The future of distributed models: model calibration and uncertainty prediction. Hydrol Process 6:279–298. doi:10.1002/hyp.3360060305

Franchini, M., 1996. Use of a genetic algorithm combined with a local search method for the automatic calibration of conceptual rainfall-runoff models. Hydrological Sciences Journal, 41(1): 21-39.

Freer J, Beven K, Ambroise B, 1996. Bayesian estimation of uncertainty in runoff prediction and the value of data: an application of the GLUE approach. Water Resour Res 32(7): 2161–2173. doi:10.1029/95WR03723

Gupta, H.V., Kling, H., Yilmaz, K.K., Martinez, G.F., 2009. Decomposition of the mean squared error and NSE performance criteria: implications for improving hydrological modeling. J. Hydrol. 377 (1), 80–91.

Gelman, A., and Rubin, D. B., 1992. Inference from iterative simulation using multiple sequences. Stat. Sci., 7(4), 457–472.

Hamraz, B., Akbarpour, A., Pourreza-Bilondi, M. and Sadeghi-Tabas, S., 2015. On the assessment of ground water parameter uncertainty over an arid aquifer. Arabian journal of geosciences, 8(12): 10759-10773.

He, J., Jones, J.W., Graham, W.D., Dukes, M.D., 2010. Influence of likelihood function choice for estimating crop model parameters using the generalized likelihood uncertainty estimation method. Agric. Syst. 103: 256–264.

Izady A, Davary K, Alizadeh A, Ziaei AN, Alipoor A, Joodavi A, Brusseau ML, 2014. A framework toward developing a groundwater conceptual model. Arab J Geosci 7:3611–3631. doi:10.1007/s12517-013-0971-9

Kavetski, D., Kuczera, G., Franks, S.W., 2006. Bayesian analysis of input uncertainty in hydrological modeling: 1. Theory. Water Resour. Res. 42: W03407, doi: 10.1029/2005WR004368.

Keesman K, van Straten G, 1989. Identification and prediction propagation of uncertainty in models with bounded noise. Int J Control 49(6): 2259–2269.

Khazaei, M.R., Zahabiyoun, B., Saghafian, B. and Ahmadi, S., 2014. Development of an automatic calibration tool using genetic algorithm for the ARNO conceptual rainfall-runoff model. Arabian Journal for Science and Engineering, 39(4), pp.2535-2549.

Khazaei, M.R., Zahabiyoun, B. and Saghafian, B., 2012. Assessment of climate change impact on floods using weather generator and continuous rainfall-runoff model. International Journal of Climatology, 32(13): 1997-2006.

Laloy, E., and Vrugt, J. A., 2012. High-dimensional posterior exploration of hydrologic models using multiple-try DREAM (ZS) and high performance computing. Water Resour. Res., 48(1): 1–18.

Laloy, E., Fasbender, D., Bielders, C.L., 2010. Parameter optimization and uncertainty analysis for plot-scale continuous modeling of runoff using a formal Bayesian approach. J. Hydrol. 380: 82–93.

Makowski, D., Wallach, D. and Tremblay, M., 2002. Using a Bayesian approach to parameter estimation; comparison of the GLUE and MCMC methods. Agronomie-Sciences des Productions Vegetales et de l'Environnement, 22(2): 191-204.

Nourali, M., Ghahraman, B., Pourreza-Bilondi, M. and Davary, K., 2016. Effect of formal and informal likelihood functions on uncertainty assessment in a single event rainfall-runoff model. Journal of Hydrology, 540: 549-564.

Pourreza-Bilondi, M., Samadi, S.Z., Akhoond-Ali, A.M. and Ghahraman, B., 2016. Reliability of Semiarid Flash Flood Modeling Using Bayesian Framework. Journal of Hydrologic Engineering, p.05016039.

Pourreza-Bilondi, M. and Samadi, S.Z., 2016. Quantifying the uncertainty of semiarid flash floods using generalized likelihood uncertainty estimation. Arabian Journal of Geosciences, 9(13): 622.

Romanowicz R, Beven KJ, Tawn J, 1994. Evaluation of predictive uncertainty in non-linear hydrological models using a Bayesian approach. In: Barnettand V, Turkman KF (eds) Statistics for the environment: water related issues. Wiley, NewYork, pp 297–317

Schoups, G., Vrugt, J. A., Fenicia, F., and Van de Giesen, N. C. 2010. Corruption of accuracy and efficiency of Markov chain Monte Carlo simulation by inaccurate numerical implementation of conceptual hydrologic models. Water Resour. Res., 46: W10530.

ter Braak, C. J. F., and Vrugt, J. A., 2008. Differential evolution Markov chain with snooker updater and fewer chains. Stat. Comput., 18(4), 435–446.

Todini, E., 1988. Il modello afllussi deflussi del fiume Amo. Relazione Generale dello studio per conto della Regione Toscana, in Tech. Report. Bologna

Todini, E., 1996. The ARNO rainfall-runoff model. J. Hydrol. 175(1–4): 339–382

Vrugt, J.A., ter Braak, C.J.F., Diks, C.G.H., Robinson, B.A., Hyman, J.M., Higdon, D., 2009a. Accelerating Markov Chain Monte Carlo simulation using self-adaptative differential evolution with randomized subspace sampling. Int. J. Nonlin. Sci. Numer. Simul. 10 (3): 273–290.

Vrugt, J.A., ter Braak, C.J.F., Gupta, H.V., Robinson, B.A., 2009b. Equifinality of formal (DREAM) and informal (GLUE) Bayesian approaches in hydrologic modeling? Stoch. Environ. Res. Risk Assess. 23 (7), 1011–1026.

Wang X, He X, Williams JR, Izaurralde RC, Atwood JD, 2005. Sensitivity and uncertainty analyses of crop yields and soil organic carbon simulated with EPIC. Trans ASAE 48(3):1041–1054. doi:10.13031/2013.18515

Zhao, R.J., 1977. Flood forecasting method for humid regions of China. East China College of Hydraulic Engineering, Nanjing