

# Assessment of interconnection between two adjacent watersheds using deterministic and fuzzy approaches

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**Abstract:** The interconnection of adjacent watersheds in case of complicated karstic systems is a significant issue in water resources development planning. In an interesting case study, two watersheds in the island of Crete are studied for investigating their interconnection and assess quantitatively the volumes of water passing from the one watershed to the other. Since the karstic system is very complicated two indirect methods were employed. In the first approach the daily rainfall – runoff model Medbasin was used for assessing a large number of formulated scenarios. In the second approach a fuzzy linear regression method was applied using the streamflow data of the recipient watershed as the dependent variable, while the precipitation in the same watershed and the outflow of the contributing watershed were the independent variables. The results show that there is a significant interconnection between the two watersheds. Useful conclusions were drawn for the implementation of the proposed methods in other similar watershed interconnection studies.

**Key words:** watershed interconnection, rainfall – runoff model, fuzzy linear regression, conceptual model, surface water potential

## 1. INTRODUCTION

The assessment of the water resources potential of a region is a very important though not easy task for formulating a management plan and select the appropriate scenario of water resources development. During the estimation of water resources potential various complications may create difficulties and uncertainties in its evaluation. Some of these complications may be introduced by existing interconnections between adjacent watersheds. A typical case of this kind is the case of karstic boundaries between the watersheds.

In this study two adjacent watersheds are considered and their interconnection is assessed. The selected basins are the Aposelemis torrent watershed and the Lassithi Plateau in the island of Crete. A qualitative experiment has shown that there is substantial interconnection between the above two watersheds (IGME, 2003). The objective of this study is to assess quantitatively the volumes of water passing through the karstic boundary from Lassithi Plateau to the recipient watershed of Aposelemis torrent.

The methods employed in the study are indirect methods avoiding to simulate the complexity of the karstic boundary. Instead, a conceptual rainfall – runoff model and a fuzzy linear regression method are utilized.

## 2. METHODOLOGY

### *2.2 Conceptual rainfall – runoff simulation model*

The first approach for assessing the interconnection between the two watersheds is the simulation of various interconnection scenarios, performed with the software package Medbasin

(Tsakiris et al., 2004; Tigkas and Tsakiris, 2004) which is based on a conceptual daily rainfall – runoff model (MERO). This model has been used extensively by FAO in Mediterranean watersheds (e.g. Underhill et al., 1970; Schenkeveld, 1971). The hydrologic cycle processes and interactions are described by empirical relationships (Giakoumakis et al., 1991). The physical characteristics of the watershed are represented by 14 parameters. Daily values of precipitation and potential evapotranspiration are used as input data, while daily and monthly streamflow values are the output of the model.

For the formulation of the interconnection scenarios the total streamflow of the recipient watershed (WQ2) is accounted as the sum of the estimated value from the rainfall – runoff (R-R) simulation model applied to the recipient watershed, together with the inflow from the contributing watershed. A number of discharge thresholds are defined in order to assess the inflow amount.

The procedure for formulating each inflow scenario is presented graphically in Figure 1. If the measured daily discharge from the contributing watershed (WQ1) is greater than the defined Threshold 1 then this is multiplied by a defined percentage (A1%) and the result is added as direct inflow to the recipient watershed. If the value of WQ1 is smaller than Threshold 1 and greater than Threshold 2 then another defined percentage (A2%) is used, etc. Another percentage (a%) of WQ1 may be added as inflow to the recipient watershed after a specified time lag (n days).

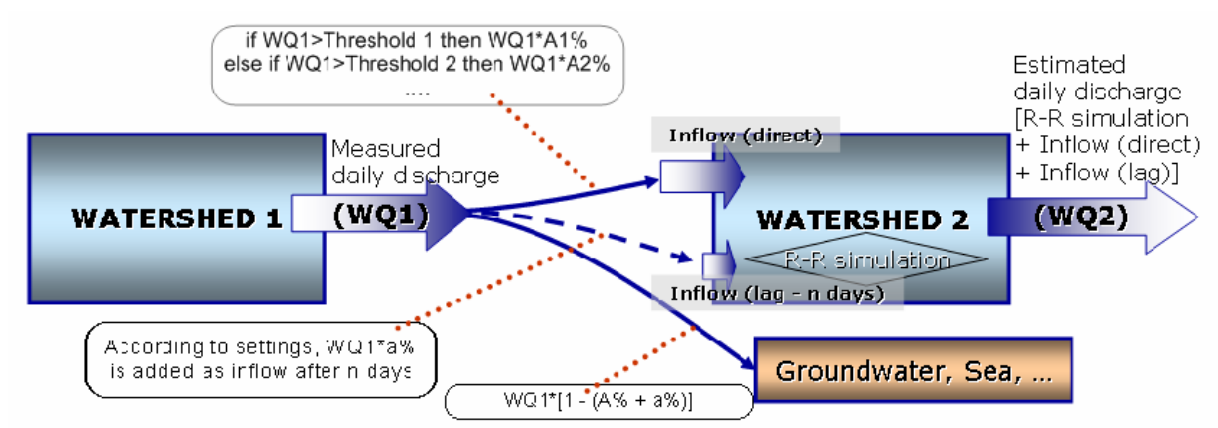


Figure 1. Formulation of inflow scenarios from the contributing watershed.

Also the pattern of the inflow can be described by a number of decay functions such of the type of those presented in Figure 2.

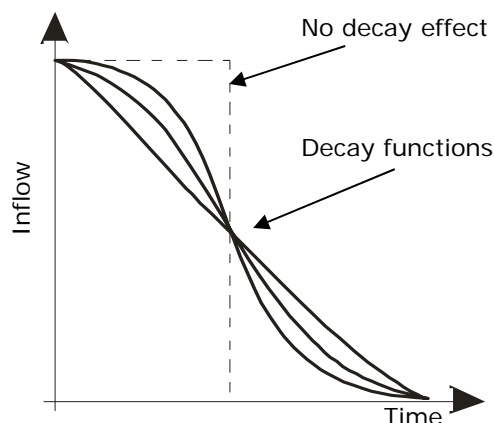


Figure 2. Use of the decay effect concept.

The assessment of performance of each scenario may be realized using a number of criteria (Nash and Sutcliffe, 1970; Cavadias and Morin, 1986; Yapo et al., 1998; WMO, 1992), the most important of which are:

a) The root mean square error of the streamflow variables (RMSE):

$$\text{RMSE} = \left( \sum [q_{\text{sim}} - q_{\text{obs}}]^2 / n \right)^{-1/2} \quad (1)$$

b) One minus the ratio of the sum of squares of the daily residuals to the sum of squares of the deviations of the observed flows from their mean (NTD):

$$\text{NTD} = 1 - \frac{\sum (q_{\text{sim}} - q_{\text{obs}})^2}{\sum (q_{\text{obs}} - \bar{q}_{\text{obs}})^2} \quad (2)$$

in which  $q_{\text{sim}}$  is the simulated discharge (including the inflow),  $q_{\text{obs}}$  is the observed discharge,  $n$  is the total number of observations and  $\bar{q}_{\text{obs}}$  is the arithmetic mean of the observed values.

## 2.2 Fuzzy linear regression model

It is known that the use of statistical linear regression is bounded by some strict assumptions related to the given data; e.g. the values of the error term should be mutually independent and identically distributed. The coefficients of the regression equations are crisp numbers.

In the fuzzy linear regression introduced by Tanaka et al. (1982; 1987 and 1989), some of the above strict assumptions of the statistical regression are relaxed. Then the regression function is a fuzzy relation.

It should be noticed that fuzzy regression may be useful in expressing functional relationships between variables, when data is not sufficient in number (Ganoulis, 1994; Peters, 1994).

In order to investigate such complex physical phenomena as the interconnection between adjacent watersheds, it seems reasonable to use the fuzzy regression approach.

The fuzzy linear regression model has the following form for each of the  $m$  data ( $j = 1(1)m$ ) (Tanaka et al., 1989):

$$Y_j = \tilde{A}_0 + \tilde{A}_1 x_{1j} + \dots + \tilde{A}_i x_{ij} + \dots + \tilde{A}_n x_{nj}, \quad (3)$$

with  $i = 1, \dots, n$

where  $n$  is the number of independent variables and  $\tilde{A}_i = (a_i, c_i)_L$  are symmetric fuzzy triangular numbers selected as coefficients, which have the following membership function:

$$\mu_{A_i}(\alpha_i) = \begin{cases} 1 - \frac{|\alpha_i - a_i|}{c_i}, & \text{if } a_i - c_i \leq \alpha_i \leq a_i + c_i \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

where  $a_i$  and  $c_i$  are the centres and the widths of the fuzzy coefficients, respectively, and  $\alpha$  is the independent variable for the membership function. Therefore, in contrast to statistical regression, fuzzy regression analysis has no error term, while uncertainty is incorporated in the model by means of fuzzy numbers.

In fuzzy linear regression, the centres and the widths of the fuzzy coefficients may be determined by solving a linear programming problem with an objective function of minimizing the total spread

of the fuzzy outputs, on condition that the given output  $y_j$  will be included in the estimated function  $Y_j$ , that is:

$$\mu_{Y_j}(y_j) \geq h, \quad \text{for } j = 1, \dots, m, \quad (5)$$

where  $\mu_{Y_j}$  is the membership function of  $Y_j$ . This definition incorporates the sense of  $h$  – cut (e.g. Klir and Yuan, 1995)

On the basis of the previous description, the fuzzy linear regression analysis is reduced to the estimation of  $A_0$  and  $A_i = (a_i, c_i)_L$ ,  $i=1, \dots, n$  that minimizes the spread of the fuzzy output (Eq. 6.a) subject to the constraints (6.b), (6.c), (6.d). The problem is therefore transformed into a linear programming problem as follows:

$$\min J = \left\{ mc_0 + \sum_{j=1}^m \sum_{i=1}^n c_i |x_{ij}| \right\} \quad (6.a)$$

subjects to:

$$y_j \geq a_0 + \sum_{i=1}^n a_i x_{ij} - (1-h)(c_0 + \sum_{i=1}^n c_i |x_{ij}|) \quad (6.b)$$

$$y_j \leq a_0 + \sum_{i=1}^n a_i x_{ij} + (1-h)(c_0 + \sum_{i=1}^n c_i |x_{ij}|) \quad (6.c)$$

$$c_i \geq 0 \quad (6.d)$$

where  $J$  is the total spread of the fuzzy output (the sum of all spreads of  $Y_j$ ).

In this paper  $h = 0$  was selected, to simplify the calculation procedure and for avoiding a very large spread. Moreover, the optimal solution for  $h \neq 0$  can be easily obtained from the optimal solution of  $h = 0$ , since the centre of  $Y_j$  remains the same, irrespective of the evaluation of the spread which can vary according to the value of  $h$  (e.g. Papadopoulos and Sirpi, 1999).

In the Tanaka's model it is assumed that all  $y$  data lie within the interval of  $Y_j$ . In other words, the bounds of the interval are determined by the 'worst' data of a given data set. This assumption can be justified if there are only a small number of samples. This can allow the analyst to decide on outliers more reliably than in the case of long data samples (Peters, 1994).

In many applications the coefficient  $\tilde{A}_0$  is difficult to interpret. On the basis of the objective function proposed by Tanaka et al. (2000) and Tsakiris and Spiliotis (2002) in the formulation of fuzzy linear programming problem, the following form for the objective function is proposed in this study, with the coefficient  $a_0$  non-positive:

$$\min \left\{ mc_0 + \sum_{j=1}^m \sum_{i=1}^n c_i |x_{ij}| + ma'_0 \right\} \quad (7.a)$$

subjects to:

$$y_j \geq -a'_0 + \sum_{i=1}^n a_i x_{ij} - (1-h)(c_0 + \sum_{i=1}^n c_i |x_{ij}|) \quad (7.b)$$

$$y_j \leq -a'_0 + \sum_{i=1}^n a_i x_{ij} + (1-h)(c_0 + \sum_{i=1}^n c_i |x_{ij}|) \quad (7.c)$$

$$c_i \geq 0 \quad (7.d)$$

$$a_0 = -a'_0 \leq 0 \quad (7.e)$$

### 3. CASE STUDY

#### 3.1 Study area

The study area includes two adjacent watersheds, namely the Lassithi Plateau and Aposelemis torrent basin, which are located in the north-eastern part of the island of Crete in Greece (Figure 3).

The Plateau is a closed, mountainous catchment with an average altitude of 1124 m and total area of 128 km<sup>2</sup>. The surface water is discharged through a sinkhole system created in limestones at the southwest end of the watershed, where the streamflow is recorded at a runoff gauging station.

Aposelemis watershed has an average altitude of 464 m, a total area of 122 km<sup>2</sup> and outlet at the Cretan Sea. The sub-basin which is studied here is located upstream the gauging station of Potamies with a corresponding area of 76.4 km<sup>2</sup>. A construction of an embankment dam is planned about one km upstream.

The main stream of Aposelemis begins from Kastamonitsa village, where a group of karstic springs are also located. These springs are directly linked with the Plateau's sinkholes, since high volumes of discharge are observed after the days of increased discharge entering the sinkholes. Therefore, the available surface water from the area cannot be calculated as a sum of the two streamflow measurements, since the streamflow of Aposelemis includes a part of Plateau's measured runoff.

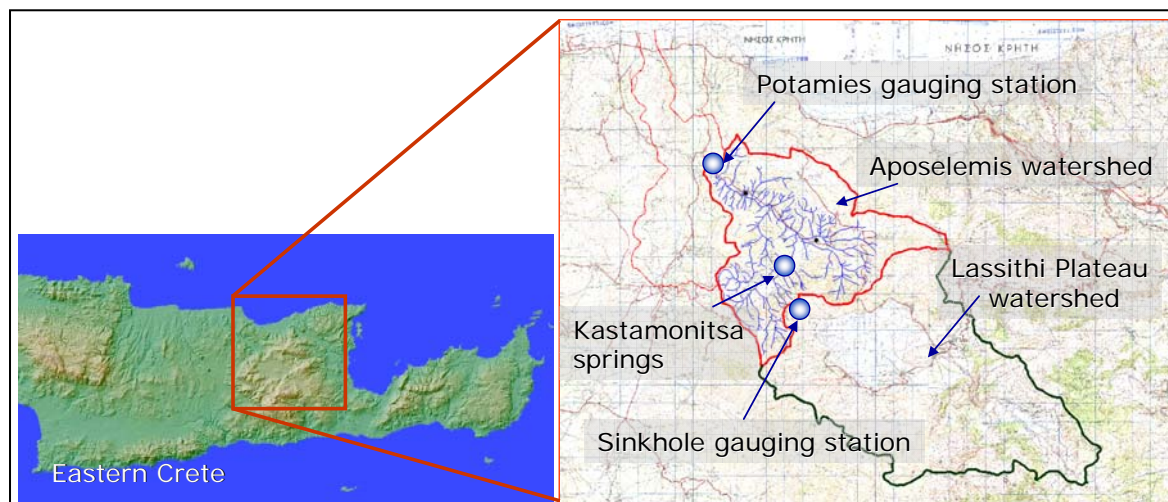


Figure 3. Study area.

#### 3.2 Application and Results

##### 3.2.1 Deterministic simulation approach

The conceptual deterministic approach for assessing the interconnection between the two watersheds was carried out by simulating several scenarios over a period of 24 hydrological years

(1973–1997) for which daily meteorological and hydrometric data were available. Rainfall and pan evaporation data from two meteorological stations (Abdou and Kasteli) were used. The pan evaporation data were transformed to potential evapotranspiration by using monthly coefficients suitable for this area (Vardavas et al., 1997). Streamflow data from Sinkhole and Potamies gauging stations were used for Plateau's and Aposelemis' runoff, respectively. The parameters of the model were assigned by the physical characteristics of the watershed.

Regarding the interconnection scenarios, various threshold levels have been tested. The thresholds that finally selected are 8, 5, 3 and 2 m<sup>3</sup>/s, respectively. The first coinciding with flood conditions near the sinkholes. The inflow to Aposelemis for each threshold level is a percentage of the total discharge from Plateau watershed. For higher discharge values the percentages for the direct inflow are also increased, as a result of the hydraulic pressure in the karstic system.

The best scenarios for inflow to the Aposelemis watershed fit within a range of 30 – 40% of the Plateau watershed's discharge in average annual terms, with optimum value around 34%. For the optimum scenario the percentages for the direct inflow are at 55%, 40%, 35% and 15% for the above mentioned threshold levels, respectively, while about 3% of runoff contributes to Aposelemis watershed with a lag of one month.

The measured and the simulated hydrographs of Aposelemis river appear in Figure 4. Both the "no inflow" and "34% inflow" scenarios are presented. It can be observed that the "34% inflow" scenario performs significantly better than the "no inflow" scenario.

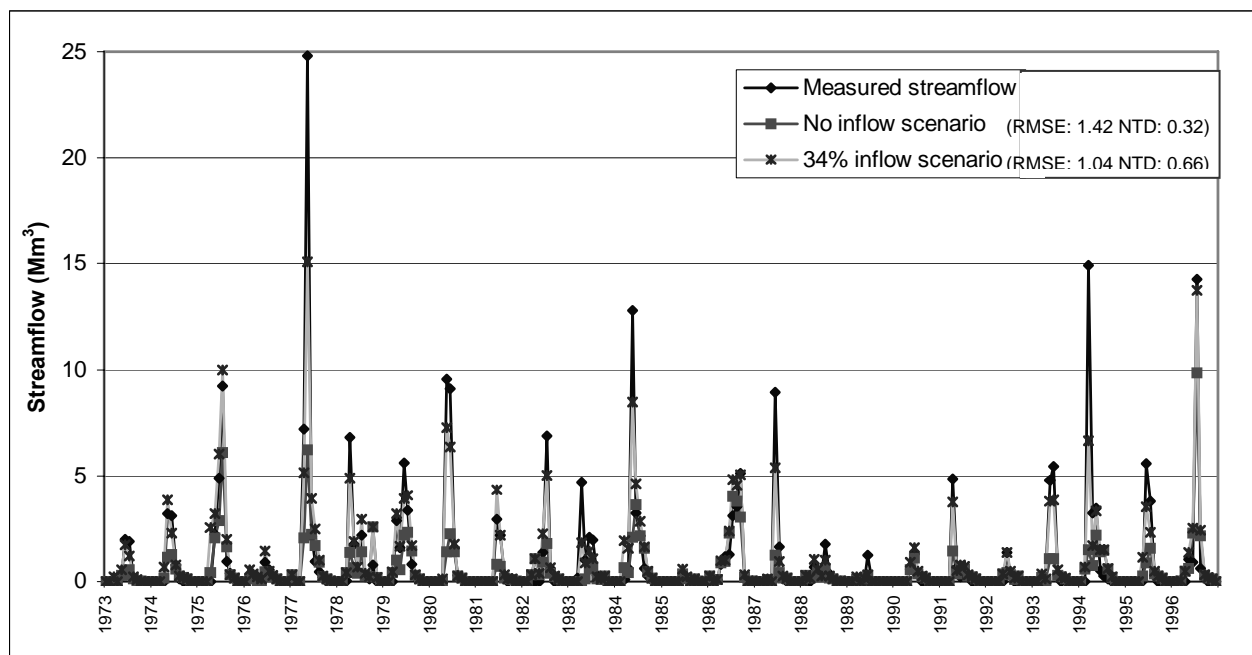


Figure 4. Measured and simulated hydrographs of Aposelemis torrent.

The distribution of the direct inflow on a monthly basis for the optimum scenario, together with the monthly runoff volumes for both watersheds, is presented in Table 1, while Figure 5 shows the percentage of the inflow in comparison with the total runoff of Plateau. If the total runoff volume of the area would be considered as the sum of Aposelemis and Plateau, this would be approximately 27.1 Mm<sup>3</sup> (mean annual runoff values). However, the real value of total runoff, from which the available surface water of the area can be estimated in order to be considered in water management plans, is significantly less, since the inflow volume must be abstracted (27.1 - 5.1 = 22 Mm<sup>3</sup>).

Table 1. Mean monthly runoff volumes at Lassithi Plateau and Aposelemis watersheds and inflow volume to Aposelemis.

Runoff (Mm <sup>3</sup> )	O	N	D	J	F	M	A	M	J	J	A	S	Annual
Plateau	0.087	0.748	2.503	4.634	4.384	3.207	0.543	0.260	0.032	0.000	0.000	0.000	<b>16.398</b>
Aposelemis	0.009	0.647	1.077	2.809	2.675	2.566	0.515	0.328	0.069	0.007	0.001	0.003	<b>10.707</b>
Direct inflow	0.024	0.286	0.898	1.566	1.350	0.818	0.084	0.091	0.001	0.000	0.000	0.000	<b>5.119</b>

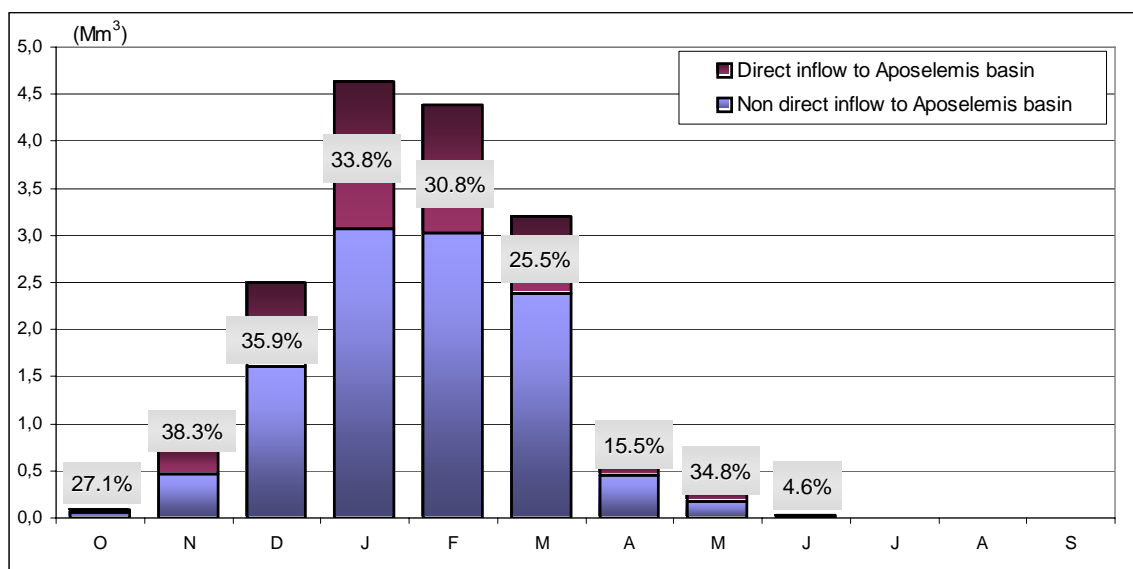


Figure 5. Mean monthly volumes of Plateau runoff.

### 3.2.2 Fuzzy approach

In the second approach, the use of fuzzy linear regression was applied in order to quantify the interconnection between the Aposelemis watershed and the Lassithi Plateau watershed. The Aposelemis streamflow was selected as the dependent variable. The precipitation of the watershed and the streamflow for the Plateau of the current and previous months were selected as independent variables.

The constraints described by Eq. 7.b, 7.c, 7.d, and 7.e for  $h = 0$  were applied for each month separately. The estimated monthly values of Aposelemis streamflow (in the form of fuzzy numbers) must therefore include all the observed monthly streamflow values. Finally the problem of fuzzy linear regression is transformed into a linear programming problem, as mentioned earlier.

The contribution of each of the independent variables (precipitation of the watershed, streamflow of Plateau of the current month, one month before and two months before) was determined with respect to the regression coefficient using the centre value.

From the results presented in Figure 6 it was concluded that a significant percentage of the streamflow of the Plateau watershed is included to Aposelemis streamflow. The major percentage of this contribution derives from Plateau's streamflow during the same month. However, there is also a considerable influence on Aposelemis streamflow caused by the Plateau's streamflow during the previous months.

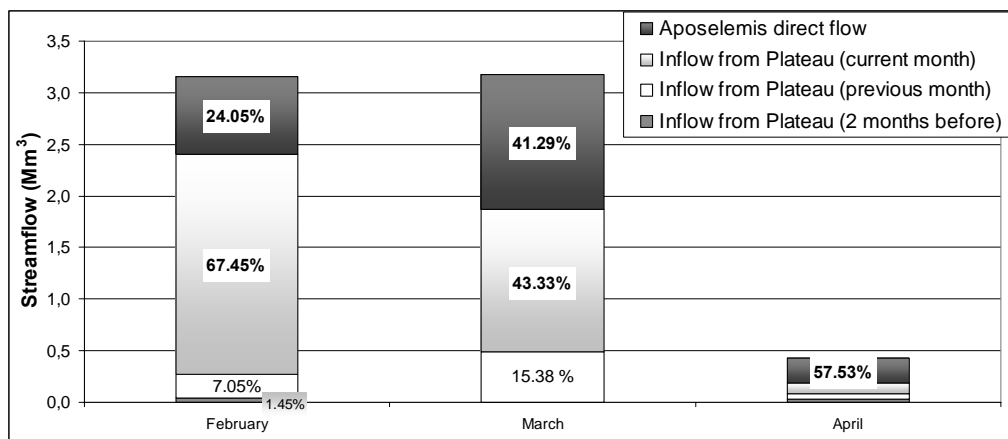


Figure 6. Monthly contributions to Aposelemis streamflow.

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